ALGEBRA I READINESS RESOURCES

STUDENT PACKET

MIAMI-DADE COUNTY PUBLIC SCHOOLS

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Summer 2021

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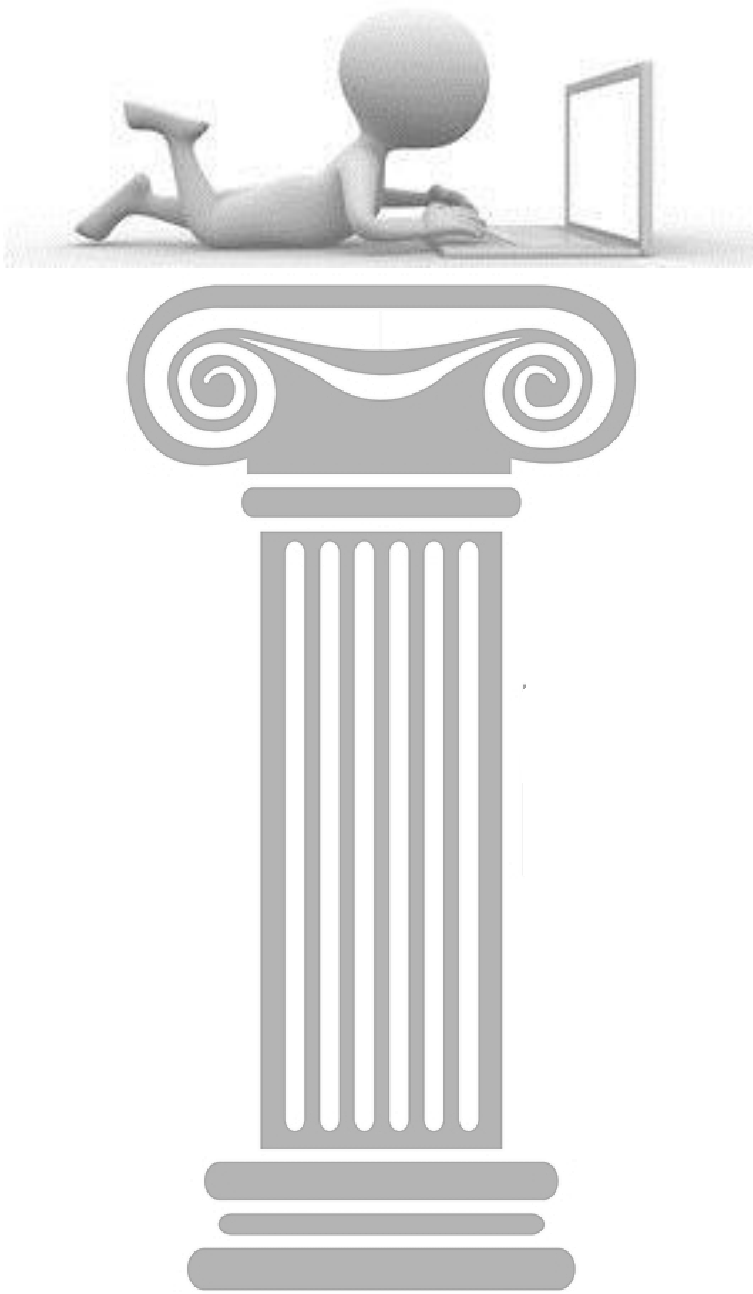
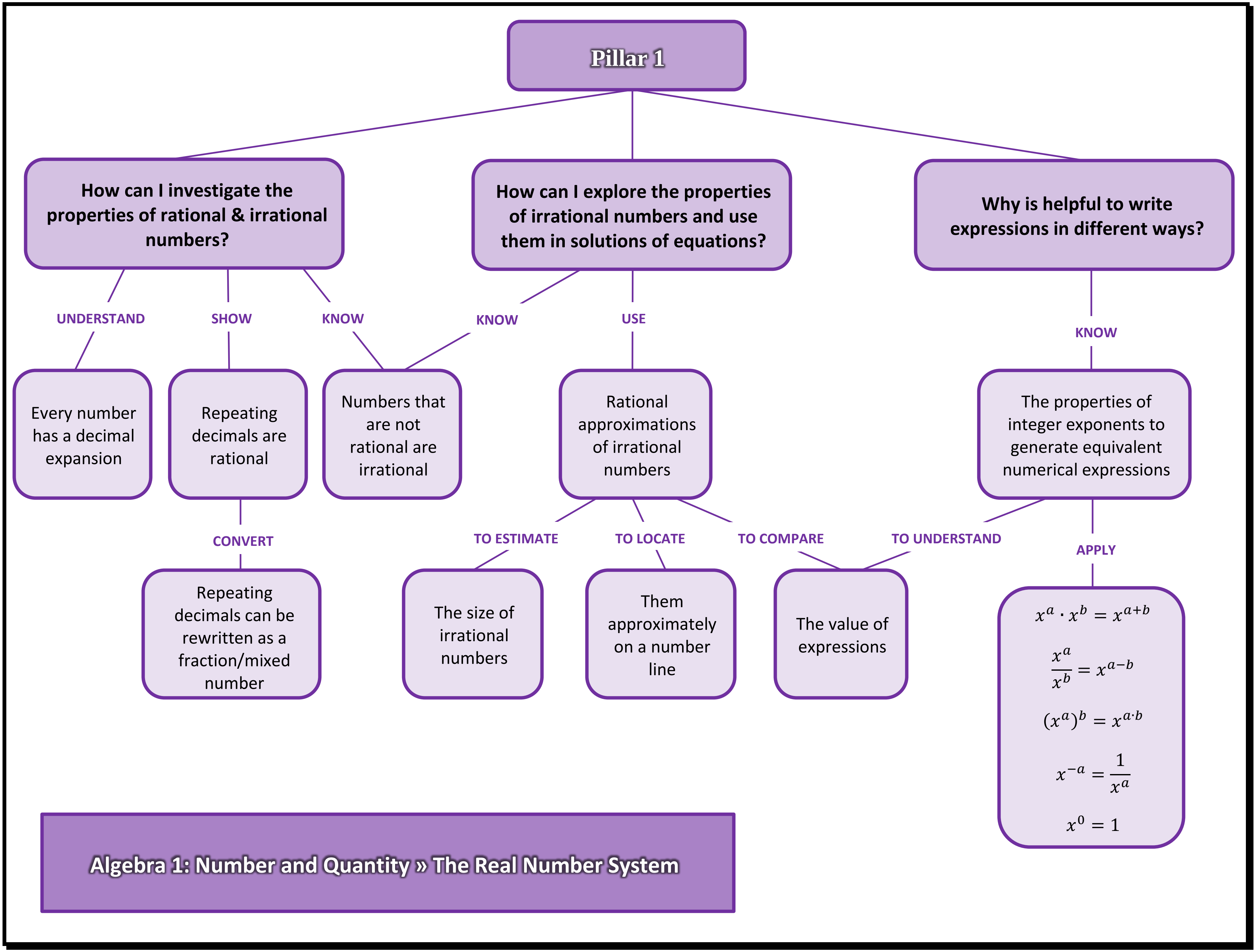


PILLAR #1



Pillar 1
How can I investigate the properties of rational & irrational numbers?
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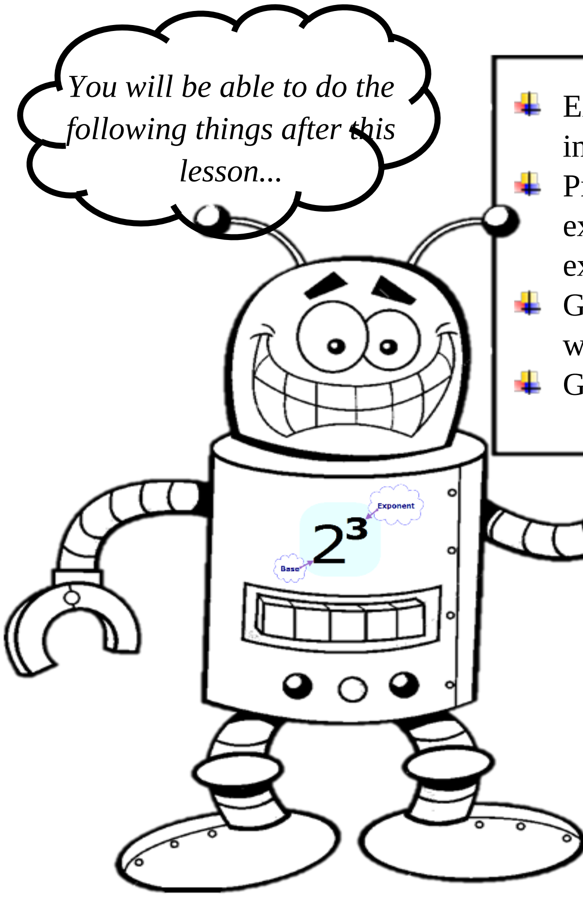
Algebra 1: Number and Quantity » The Real Number System

Lesson 1: Integer Exponents



- Expand, simplify, and evaluate expressions involving exponents, including products and quotients raised to powers.
- Prove the rules of exponents for multiplying and dividing exponents with the same base by using the definition of an exponent
- Generate and use the rules for multiplying and dividing powers with the same base
- Generate and use the rules for zero and negative exponents

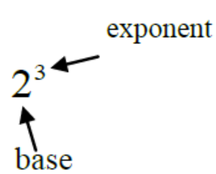
Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- Base
- Exponent
- Power
- Order of Operation
- Scientific Notation
- Magnitude

Integer Exponents

An *exponent* is the superscript which tells how many times the base is used as a factor.



In the number 2^3, read "2 to the third power" or "2 cubed", the 2 is called the *base* and the 3 is called the *exponent*.

Examples:

$2^3 = 2 \cdot 2 \cdot 2$

$5^2 = 5 \cdot 5$

$6^4 = 6 \cdot 6 \cdot 6 \cdot 6$

To write an exponential in standard form, compute the products. i.e. $5^2 = 5 \cdot 5 = 25$

Example: Powers of 10 in scientific notation,

$10 = 10$

$10^2 = 100$

$10^3 = 1000$

What pattern allows you to find the value of an exponential with base 10 quickly?

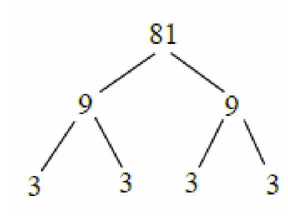
Answer: The number of zeroes is equal to the exponent!

Caution: If a number does not have an exponent visible, it is understood to have an exponent of ONE!

Let's practice writing numbers in exponential form.

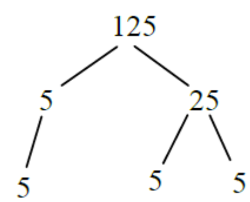
Examples: Write 81 with a base of 3.

$\mathbf{81 = 3^?}$, $\mathbf{81 = 3 \cdot 3 \cdot 3 \cdot 3}$, therefore $\mathbf{81 = 3^4}$



Write 125 with a base of 5.

$\mathbf{125 = 5^?, 125 = 5 \cdot 5 \cdot 5, \mathit{therefore}\ 125 = 5^3}$



In algebra, we often must find the products and quotients of algebraic expression. For example, what is the product of the problem below?

Rewrite each term in expanded form, and then convert it back to exponential form.

Since $x^3 = x \cdot x \cdot x$

and $x^2 = x \cdot x$

$$x^3 \cdot x^2 = (x \cdot x \cdot x) \cdot (x \cdot x) \text{ or } x^5.$$

Caution: If you jump to an answer of x^6, this is incorrect. Watch for this error!

We do not multiply the exponents as we might suspect: we ***Add*** them! Let's try a few more problems to verify our conjecture:

Examples:

$x^4 \cdot x^3 = (x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x)$ *or* x^7 *i.e.,* $x^4 \cdot x^3 = x^{4+3}$ *or* x^7

$x \cdot x^5 = x \cdot (x \cdot x \cdot x \cdot x \cdot x)$ *or* x^6 *i.e.,* $x \cdot x^5 = x^{1+5}$ *or* x^6

We are now ready to state the rule for multiplying exponential expressions with the same base.

> *When multiplying powers with the same base,* ***add*** *their exponents;*
> *that is* $x^a \cdot x^b = x^{a+b}$

Check Yourself: Simplify.

1. $5^3 \cdot 5^7$
2. $t^4 \cdot t^6$
3. $5^2 \cdot 5^8$
4. $b^{10} \cdot b^2$

What might we suspect about the rule for division?

Since division is the inverse of multiplication, and multiplying exponential expressions involves the addition of exponents, what would division of exponential expressions involve? We might suggest subtraction is the key here; we can show this to be true with a few examples:

Example:

$\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = x \cdot x \cdot x = x^3$ $\qquad$ $i.e. \frac{x^5}{x^2} = x^{5-2}$ or x^3

Example:

$\frac{x^6}{x} = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x} = x \cdot x \cdot x \cdot x \cdot x = x^5$ $\qquad$ $i.e. \frac{x^6}{x} = x^{6-1}$ or x^5

Let's now state the rule:

> *When dividing powers with the same base,* ***subtract*** *their exponents*
> *(subtract the exponent in the denominator from the exponent in the numerator);*
> *that is,* $\frac{x^a}{x^b} = x^{a-b}$

Check Yourself: Simplify.

1. $\frac{f^5}{f^2}$
2. $\frac{u^{11}}{u^4}$
3. $\frac{5^8}{5}$
4. $\frac{4^{12}}{4^7}$

▽ Be careful with your integer operations —some students have the tendency to add when they should multiply. A simple example is to look at the following product: $2x^3 \cdot 4x^4$. The answer is $8x^7$, of course. Common errors are to multiply all numbers involved, arriving at the incorrect answer of $8x^{12}$; or to add all numbers, arriving at the incorrect answer of $6x^7$.

Look at the following example for other errors to watch for.

Example: $\frac{8x^6}{4x^3} = 2x^3$;

***Common Errors*:**

→ Subtract everything: $4x^3$

→ Divide everything: $2x^2$

Example: $3a^5 \cdot 5a^2 = 15a^7$;

Common Errors:

→ Add everything: $8a^7$

→ Multiply everything: $15a^{10}$

At this point, you may be feeling that math rules always make sense! Remember you can always go back to expanded notation and arrive to an answer without using the rules for exponents.

To see what happens when you raise a power to a power, use the order of operations.

$(5^3)^2 = (5 \cdot 5 \cdot 5)^2$	Evaluate the power inside the parentheses.
$= (5 \cdot 5 \cdot 5)(5 \cdot 5 \cdot 5)$	Evaluate the power outside the parentheses.
$= 5^6$	

Let’s now state the rule:

> *When raising a power to a power, keep the base and* ***multiply*** *the exponents;*
>
> *that is,* $(x^a)^b = x^{a \cdot b}$

Examples:

$(7^3)^5 = 7^{3 \cdot 5} = 7^{15}$

$(5^{10})^{-6} = 5^{10(-6)} = 5^{-60}$

$(3^{-7})^{-2} = 3^{-7(-2)} = 3^{14}$

Check Yourself: Simplify.

1. $(5^4)^5$
2. $(a^3)^7$
3. $(r^6)^5$
4. $(3^2)^9$

Negative and Zero Exponents

Pattern development is a very effective way to learn the concept of negative and zero exponents. Consider the following pattern that you should have seen previously.

$$\begin{aligned} 2^4 &= 2\cdot 2\cdot 2\cdot 2 = 16 \\ & \qquad \div 2 \\ 2^3 &= 2\cdot 2\cdot 2 \quad = 8 \\ & \qquad \div 2 \\ 2^2 &= 2\cdot 2 \quad = 4 \\ & \qquad \div 2 \\ 2^1 &= 2 \quad = 2 \\ & \qquad \div 2 \\ 2^0 &= \quad = 1 \end{aligned}$$

As we review this pattern, you should see that each time the exponent is decreased by 1, the expanded form contains one less factor of 2 and the product is half of the preceding product.

$$\begin{aligned} 2^4 &= 2\cdot 2\cdot 2\cdot 2 = 16 \\ & \qquad \div 2 \\ 2^3 &= 2\cdot 2\cdot 2 \quad = 8 \\ & \qquad \div 2 \\ 2^2 &= 2\cdot 2 \quad = 4 \\ & \qquad \div 2 \\ 2^1 &= 2 \quad = 2 \\ & \qquad \div 2 \\ 2^0 &= \quad = 1 \\ & \qquad \div 2 \\ 2^{-1} &= \quad = \\ & \qquad \div 2 \\ 2^{-2} &= \quad = \end{aligned}$$

Following this pattern, $1 \div 2$ is $\frac{1}{2}$.

Continuing this pattern, $\frac{1}{2} \div 2 = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2^2} = \frac{1}{4}$

So $2^{-1} = \frac{1}{2^1}$, and $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$.

Let's now state the rule:

A base raised to the zero power equals 1
that is, $a^0 = 1$

Looking at powers of 10,

$$10^{-1} = \frac{1}{10^1} = \frac{1}{10} \ \ or \ \ 0.1$$

$$10^{-2} = \frac{1}{10^2} = \frac{1}{10 \cdot 10} = \frac{1}{100} \ \ or \ \ 0.01$$

$$10^{-3} = \frac{1}{10^3} = \frac{1}{10 \cdot 10 \cdot 10} = \frac{1}{1{,}000} \ \ or \ \ 0.001$$

Let's now state the rule:

> *A negative exponent means you must re-write our power as a fraction*
>
> *that is,* $a^{-n} = \frac{1}{a^n}$

Check Yourself: Write your answer as a base with a positive exponent.

1. a^{-6}
2. 2^{-3}
3. 6^{-2}
4. b^{-7}

Now that we know negative exponents mean reciprocal, we can perform operations with negative exponents just like we did with positive exponents. Consider the following example of the multiplication rule. Notice that we still added the exponents, but just need to write our answer as a fraction if we have a negative exponent left after multiplication.

$$(5^3)(5^{-5}) = 5^{3+(-5)} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$(4^7)(4^{-5}) = 4^{7+(-5)} = 4^2 = 16$$

Now let's look at a division example. Remember that we found we can subtract the exponents if we have the same base.

$$\frac{5^2}{5^{-2}} = 5^{2-(-2)} = 5^4 = 625$$

$$\frac{4^{-1}}{4^3} = 4^{-1-(3)} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$$

Finally, we can see that the power to a power still works with negative exponents. We simply multiply the exponents.

$$(2^3)^{-2} = 2^{3\cdot(-2)} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$$

$$(3^{-2})^{-2} = 3^{-2\cdot(-2)} = 3^4 = 81$$

Check Yourself: Write your answer as a base with a positive exponent.

1. $\dfrac{t^5}{t^{-4}}$
2. $(2^{-3})^{-5}$
3. $\dfrac{6^{-2}}{6^{-3}}$
4. $(3^{-7})^2$

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- EVALUATE EXPRESSIONS WITH EXPONENTS
- KNOW AND APPLY THE PROPERTIES OF INTEGER EXPONENTS TO GENERATE EQUIVALENT NUMERICAL EXPRESSIONS

TAKE THE **MAFS.8.EE.1.1** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.EE.1.1**

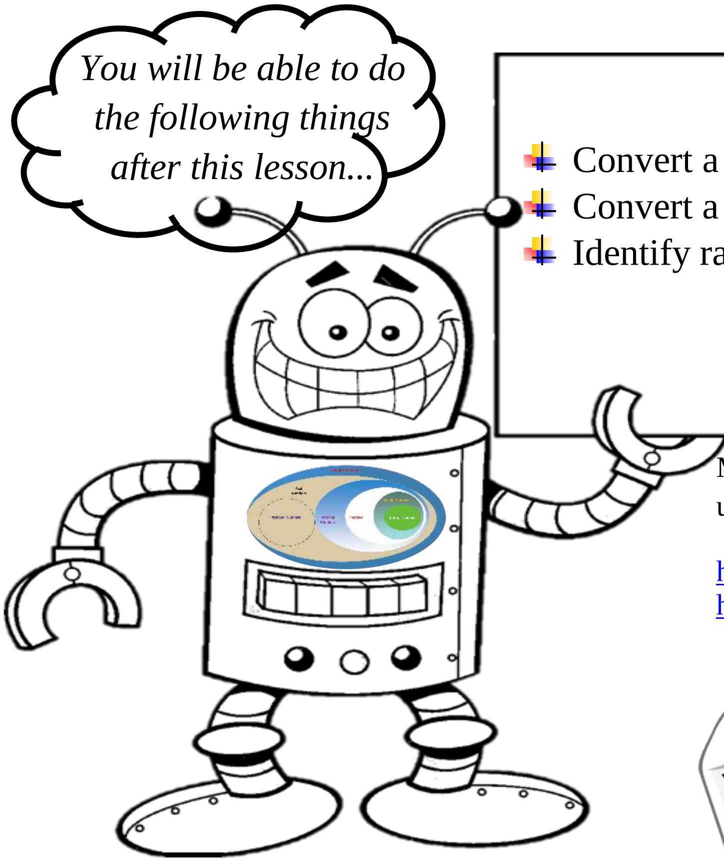
I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

- COMPLETE LESSON **MAFS.8.EE.1.1** ON EDGENUITY
- GO TO LESSON 2: IDENTIFYING RATIONAL AND IRRATIONAL NUMBERS

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 2: IDENTIFYING RATIONAL AND IRRATIONAL NUMBERS

Lesson 2: Identifying Rational and Irrational Numbers



- Convert a repeating decimal into a rational number.
- Convert a fraction into a repeating decimal.
- Identify rational and irrational numbers.

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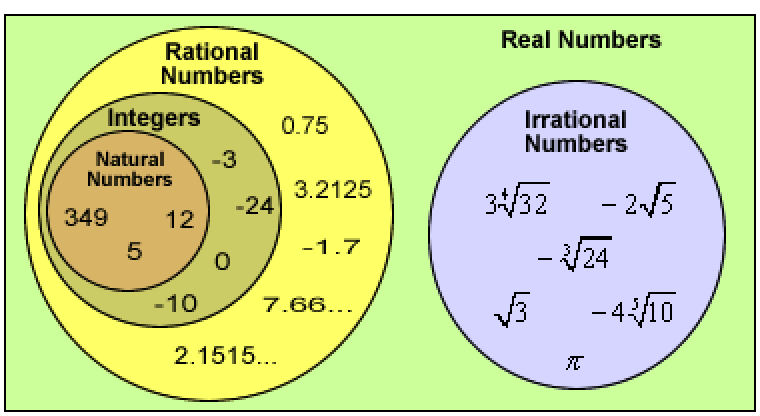
http://www.mathwords.com/

- Irrational number
- Rational number

The Real Number System

The set of *real numbers* consists of all rational and irrational numbers. The relationship can be shown in the Venn diagram below.



A *rational number* is a number that can be written as a quotient of two integers. The decimal form repeats or terminates.

An *irrational number* is a number that cannot be written as a quotient of two integers. The decimal form neither terminates nor repeats.

Rational Numbers

Rational numbers are numbers that can be written as a quotient of two integers.

Since decimals are special fractions, all the rules we learn for fractions will work for decimals. The only difference is the denominators for decimals are powers of 10; i.e., 10^1, 10^2, 10^3, 10^4, etc.... Students normally think of powers of 10 in standard form: 10, 100, 1000, 10,000, etc.

In a decimal, the numerator is the number to the right of the decimal point. The denominator is not written, but is implied by the number of digits to the right of the decimal point. The number of digits to the right of the decimal point is the same as the number of zeros in the power of 10: 10, 100, 1000, 10,000…

Therefore, one place is tenths, two places are hundredths, and three places are thousandths.

Examples:

1) 0.56 2 places $\rightarrow \frac{56}{100}$

2) 0.532 3 places $\rightarrow \frac{532}{1000}$

3) 3.2 1 place $\rightarrow 3\frac{2}{10}$

The correct way to say a decimal numeral is as follows:

1) Say the number.
2) Then say its denominator and add the suffix "*ths*".

Examples:

- 0.53 Fifty-three hundredths
- 0.702 Seven hundred two thousandths
- 0.2 Two tenths
- 0.013 Thirteen thousandths

When there are numbers, different than zero, on both sides of the decimal point, the decimal point is read as "and". You say the number on the left side of the decimal point, and then the decimal point is read as "and". You then say the number on the right side with its denominator.

Examples: 1) *Write 15.203 in word form.*

Fifteen and two hundred three thousandths

2) *Write 7.0483 in word form*.
Seven and four hundred eighty-three ten-thousandths

3) Write 247.45 in word form.

Two hundred forty-seven and forty-five hundredths

Converting Fractions to Decimals: Terminating and Repeating Decimals

A *rational number*, a number that can be written in the form of $\frac{a}{b}$ (quotient of two integers), will either be a terminating or repeating decimal. A *terminating decimal* has a finite number of decimal places; you will obtain a remainder of zero. A *repeating decimal* has a digit or a block of digits that repeat without end.

One way to convert fractions to decimals is by making equivalent fractions.

Example: *Convert* $\frac{1}{2}$ *to a decimal.*

Since a decimal is a fraction whose denominator is a power of 10, look for a power of 10 that 2 will divide into evenly.

$$\frac{1}{2} = \frac{5}{10}$$

Since the denominator is 10, we need only one digit to the right of the decimal point, and the answer is 0.5.

Example: *Convert* $\frac{3}{4}$ *to a decimal.*

Again, since a decimal is a fraction whose denominator is a power of 10, we look for powers of 10 that the denominator will divide into evenly. 4 won't go into 10, but 4 will go into 100 evenly.

$$\frac{3}{4} = \frac{75}{100}$$

Since the denominator is 100, we need two digits to the right of the decimal point, and the answer is 0.75.

There are denominators that will never divide into any power of 10 evenly. Since that happens, we look for an alternative way of converting fractions to decimals. Could you recognize numbers that are not factors of powers of ten? Using your Rules of Divisibility, factors of powers of ten can only have prime factors of 2 or 5. That would mean 12, whose prime factors are 2 and

3, would not be a factor of a power of ten. That means that 12 will never divide into a power of 10 evenly. For example, a fraction such as $\frac{5}{12}$ will not terminate – it will be a repeating decimal.

Not all fractions can be written with a power of 10 as the denominator. We need to look at another way to convert a fraction to a decimal: *divide the numerator by the denominator*.

Example: *Convert $\frac{3}{8}$ to a decimal.*

This could be done by equivalent fractions since the only prime factor of 8 is 2.

$$\frac{3}{8} \rightarrow \frac{3}{8} \cdot \frac{125}{125} = \frac{375}{1000}$$

However, it could also be done by $8\overline{)3.000}$ (quotient 0.375) division.

Doing this division problem, we get 0.375 as the equivalent decimal.

Example: *Convert $\frac{5}{12}$ to a decimal.*

This could not be done by equivalent fractions since one of the factors of 12 is 3. We can still convert it to a decimal by division.

$$\begin{array}{r} 0.41666... \\ 12\overline{)5.00000} \end{array}$$

Six is repeating, so we can write it as $0.41\overline{6}$.

The *vinculum* is written over the digit or digits that repeat.

> ***Vinculum****: A horizontal line placed above multiple quantities to indicate that they form a unit. It is commonly used to denote repeating decimals* $(e.g. 0.\overline{3})$.

Example: *Convert $\frac{4}{11}$ to a decimal.*

This would be done by division.

$$\frac{4}{11} \rightarrow \begin{array}{r} 0.3636... \\ 11\overline{)4.0000} \end{array} \text{ or } 0.\overline{36}$$

Check Yourself: Convert the following fractions to repeating decimals.

1) $\frac{11}{12}$ 2) $\frac{1}{3}$ 3) $\frac{5}{6}$ 4) $\frac{5}{11}$ 5) $\frac{1}{6}$ 6) $\frac{7}{18}$

Converting Decimals to Fractions

To convert a decimal to a fraction:

1) Determine the denominator by counting the number of digits to the right of the decimal point.
2) The numerator is the number to the right of the decimal point.
3) Simplify, if possible.

Examples: *1) Convert 0.52 to a fraction.*

$$0.52 = \frac{52}{100}$$

$$= \frac{13}{25}$$

2) *Convert 0.613 to a fraction*.

$$0.613 = \frac{613}{1000}$$

3) *Convert 8.32 to a mixed number and improper fraction.*

$$8.32 = 8\frac{32}{100}$$

$$= 8\frac{8}{25} \text{ or } \frac{208}{25}$$

But what if we have a repeating decimal?

While the decimals 0.3 and $0.\overline{3}$ look alike at first glance, they are different. They do not have the same value. We know 0.3 is three tenths, $\frac{3}{10}$. How can we say or write $0.\overline{3}$ as a fraction?

As we often do in math, we take something we don't recognize and make it look like a problem we have seen/done before. To do this, we eliminate the repeating part – the vinculum (line over the 3).

Example: *Convert* $0.\overline{3}$ *to a fraction.*

$$0.\overline{3} = 0.333333...$$

Let x = 0.333333...

Notice, this is important, that only *one number is repeating*. If we multiply both sides of the equation above by 10 (*one zero*), then subtract the two equations, the repeating part disappears.

$$
\begin{aligned}
10x &= 3.3333\ldots \\
-\ \ x &= 0.3333\ldots \\
\hline
9x &= 3.0000 \\
\frac{9x}{9} &= \frac{3.0000}{9} \\
x &= \frac{1}{3}
\end{aligned}
\qquad \frac{1}{3} \text{ is the equivalent fraction for } 0.\overline{3}
$$

Example: *Convert* $0.\overline{345}$ *to a fraction.*

The difficulty with this problem is the decimal is repeating. So, we eliminate the repeating part by letting

$$x = 0.345345345\ \ldots$$

$$0.\overline{345} = 0.345345345\ldots$$

$$
\begin{aligned}
1000x &= 345.345345345\ldots \\
-\ \ x &= \ \ 0.345345345\ldots \\
\hline
\frac{999x}{999} &= \frac{345}{999} \\
x &= \frac{345}{999} \text{ or } \frac{115}{333}
\end{aligned}
$$

Note, *three digits are repeating*. By multiplying both sides of the equation by 1000 (*three zeros*), the repeating parts line up. When we subtract, the repeating part disappears.

Example: *Convert* $0.1\overline{3}$ *to a fraction.*

Note, *one digit is repeating, but one is not*. By multiplying both sides of the equation by 10, the repeating parts line up. When we subtract, the repeating part disappears.

$$
\begin{aligned}
10x &= 1.33333\ldots \\
-\ x &= \ \ .13333\ldots \\
\hline
9x &= 1.2 \\
\frac{9x}{9} &= \frac{1.2}{9} \\
x &= \frac{1.2}{9} \quad \text{or} \quad \frac{12}{90} \left(\text{which simplifies to} \frac{2}{15}\right)
\end{aligned}
$$

Ready for a "short cut"? Let's look at some patterns for repeating decimals.

$\frac{1}{9} = 0.111\ldots$ or $0.\overline{1}$ $\frac{1}{11} = 0.0909\ldots$ or $0.\overline{09}$

$\frac{2}{9} = 0.222\ldots$ or $0.\overline{2}$ $\frac{2}{11} = 0.1818\ldots$ or $0.\overline{18}$

$\frac{3}{9} = 0.333\ldots$ or ? $\frac{3}{11} = 0.2727\ldots$ or ?

$\frac{4}{9} =$? $\frac{4}{11} =$?

It is easy to generate the missing decimals when you see the pattern!

Let's continue to look at a few more repeating decimals, converting back into fractional form.

Because we are concentrating on the pattern, we will choose NOT to simplify fractions where applicable. This would be a step to add later.

$0.\overline{5} = \frac{5}{9}$	$0.\overline{13} = \frac{13}{99}$	$0.\overline{123} = \frac{123}{999}$
$0.\overline{6} = \frac{6}{9}$	$0.\overline{25} = \frac{25}{99}$	$0.\overline{154} = \frac{154}{999}$
$0.\overline{7} = \frac{?}{9}$	$0.\overline{37} = \frac{?}{99}$	$0.\overline{421} = \frac{?}{999}$
$0.\overline{8} = \frac{?}{?}$	$0.\overline{56} = \frac{?}{?}$	$0.\overline{563} = \frac{?}{?}$

The numerator of the fraction is the same numeral as the numeral under the vinculum. We can also quickly determine the denominator: it is *9ths* for *one* place under the vinculum, *99ths* for *two* places under the vinculum, *999ths* for *three* places under the vinculum, and so on.

But what if the decimal is of a form where not all the numerals are under the vinculum? Let's look at a few.

$$0.2\overline{3} = \frac{21}{90}$$

$$0.3\overline{5} = \frac{32}{90}$$

$$0.4\overline{27} = \frac{423}{990}$$

$$0.3\overline{25} = \frac{322}{990}$$

$$0.4\overline{276} = \frac{4272}{9990}$$

$$0.23\overline{5} = \frac{212}{900}$$

The numerator is generated by subtracting the number *not* under the vinculum from the entire number (including the digits under the vinculum).

We still determine the number of nines in the denominator by looking at the number of digits under the vinculum. The number of digits *not* under the vinculum gives us the number of zeroes.

$$0.37\overline{59} = \frac{3722}{9900}$$

$$0.42\overline{76} = \frac{4234}{9900}$$

$$0.201\overline{5} = \frac{1814}{9000}$$

$$0.60\overline{24} = \frac{5964}{9900}$$

$$0.814\overline{37} = \frac{80623}{99000}$$

$$0.5534\overline{1} = \frac{49807}{90000}$$

Note that again we chose not to simplify fractions where applicable as we want to concentrate on the pattern.

Does $0.\overline{9} = 1$??

Do you believe it? Let's look at some reasons why it's true. Using the method we just looked at:

$$\begin{array}{rrl} & 10x = & 9.9999999\ldots \\ - & x = & .9999999\ldots \\ \hline & 9x = & 9 \end{array}$$

Surely if $9x = 9$, then $x = 1$. But since x also equals $0.9999999\ldots$ we get that $0.9999999\ldots = 1$.

But this is unconvincing to many people. So, here's another argument. Most people who have trouble with this fact oddly *don't* have trouble with the fact that $\frac{1}{3} = 0.3333\ldots$. Well, consider the following addition of equations then:

$$\begin{array}{rl} & \frac{1}{3} = .3333333\ldots \\ + & \frac{2}{3} = .6666666\ldots \\ \hline & \frac{3}{3} = .9999999\ldots \end{array}$$

This seems simplistic, but it's very, very convincing, isn't it? Or try it with some other denominator:

$$\begin{array}{rl} & \frac{1}{11} = .09090909\ldots \\ + & \frac{10}{11} = .90909090\ldots \\ \hline & \frac{11}{11} = .99999999\ldots \end{array}$$

Which works out very nicely. Or even:

$$\begin{array}{rl} & \frac{2}{7} = .285714285714\ldots \\ + & \frac{5}{7} = .714285714285\ldots \\ \hline & \frac{7}{7} = .999999999999\ldots \end{array}$$

It will work for any two fractions that have a repeating decimal representation and that add up to 1. The problem, though, is BELIEVING it is true.

So, you might think of 0.9999.... as another name for 1, just as 0.333... is another name for 1/3.

Check Yourself: Convert the following decimals to fractions

1. $0.\overline{2}$
2. $0.\overline{15}$
3. $0.\overline{36}$
4. $0.\overline{215}$
5. $1.2\overline{3}$
6. $3.\overline{25}$

Irrational Numbers

Recall that rational numbers are any number that can be expressed as a fraction where the numerator and denominator are both integers. Sometimes you will see this fraction written as $\frac{\mathrm{p}}{\mathrm{q}}$ where p and q are both integers.

You might recall that repeating decimals, decimals that follow a repeating numeric pattern, are rational numbers. For example, $0.08\overline{3} = \frac{5}{6}$ and $0.\overline{142857} = \frac{1}{7}$. Remember that even terminating decimals, meaning decimals that stop, are really repeating decimals and therefore rational. For example, $0.75 = 0.75000000000000\ldots$ which shows that the zero is repeating meaning 0.75 is rational. In fact, it is equal to $\frac{3}{4}$.

All of this helps us to define irrational numbers. The prefix *ir-* means "not" so that we can define irrational numbers as *numbers that are not rational*. In other words, an irrational number cannot be written as a fraction. An irrational number written as a decimal would go forever and have no repeating pattern.

The most common example of this is the number π which you may know that it is approximately 3.14 or $\frac{22}{7}$. However, both of those values are only rational estimates of π. Other than a few special numbers like π or *e* (which you'll learn about in later math courses), irrational numbers come up most often when dealing with square roots.

Recall that a square root is the inverse operation of squaring a number. In other words, we are asking ourselves, "What number multiplied by itself will equal the given number?". The symbol for square root is $\sqrt{\ }$ and you should remember some basics such as $\sqrt{25} = 5$ or $\sqrt{0.49} = 0.7$ when we take the principal (or positive) square root.

When square roots don't have exact solutions such as the examples above, they are irrational. So, all the following are irrational numbers because they don't have an exact solution: $\sqrt{60}, \sqrt{11}, \sqrt{2}, \sqrt{77}, \sqrt{21}$. Note that $\sqrt{2}$ is irrational. This is something you should memorize.

Check Yourself: Rational or Irrational?

Directions: Shade in the irrational numbers to find your path from the first row to the last row.

$\frac{2}{3}$	0.25	$\sqrt{56}$	$\sqrt{36}$	0.6	-3	$\sqrt{225}$	$-0.\bar{4}$	$\sqrt[3]{8}$	$\sqrt{196}$
$5\frac{3}{4}$	$\frac{5}{3}$	$\sqrt{15}$	$2\sqrt{49}$	50%	$-2\frac{1}{4}$	3π	0.576…	0.378…	-45%
$3\sqrt{36}$	0	π	0.123…	$\sqrt{65}$	0.284…	$-\sqrt{21}$	$\sqrt{1}$	$3\sqrt{75}$	-78
0 + -4	$\sqrt[3]{27}$	6.37	$8\frac{5}{7}$	$\sqrt{9}$	$\frac{2}{5}$	$8.\bar{5}$	-3	$6\sqrt{15}$	$\sqrt{4}$
$2\sqrt{16}$	$\pi + 4$	$\sqrt{95}$	$-\sqrt{63}$	0.245…	$\sqrt{5}$	37%	$\sqrt{81}$	$\frac{\pi}{2}$	0.702
$-5 + \frac{1}{4}$	0.731…	$0.\bar{8}$	$2+\sqrt{36}$	0.245	$\sqrt{48}$	π^5	0.598…	$\sqrt{35}$	$\sqrt{100}$
75%	$\sqrt{10}$	$3 - \pi$	$\sqrt{99}$	$\sqrt[3]{64}$	$\frac{1}{3}$	$\sqrt{121}$	$-3\frac{2}{5}$	$\frac{6}{7}$	-45%
$\sqrt{400}$	1 - 0.6	7^3	$\sqrt{\pi}$	0.967…	$\sqrt{31}$	$-\sqrt{80}$	$6 + \frac{1}{8}$	$\sqrt{49}$	$\sqrt[3]{125}$
$(-23)^5$	$\frac{7}{9}$	56 ÷8	$\sqrt{169}$	$5\frac{3}{7}$	56%	$\pi - 5$	$\sqrt{4\pi}$	$\sqrt{37}$	$0.\overline{35}$
-6.89	4 + -3	5 · -2	-69	$\frac{9}{10}$	$2\sqrt{16}$	23%	0.142	0.142…	$\sqrt{64}$

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- [UNDERSTAND RATIONAL AND IRRATIONAL NUMBERS]
- [CONVERTING REPEATING DECIMALS INTO FRACTIONS]
- [DISTINGUISH BETWEEN RATIONAL AND IRRATIONAL NUMBERS]

TAKE THE **MAFS.8.NS.1.1** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.NS.1.1**

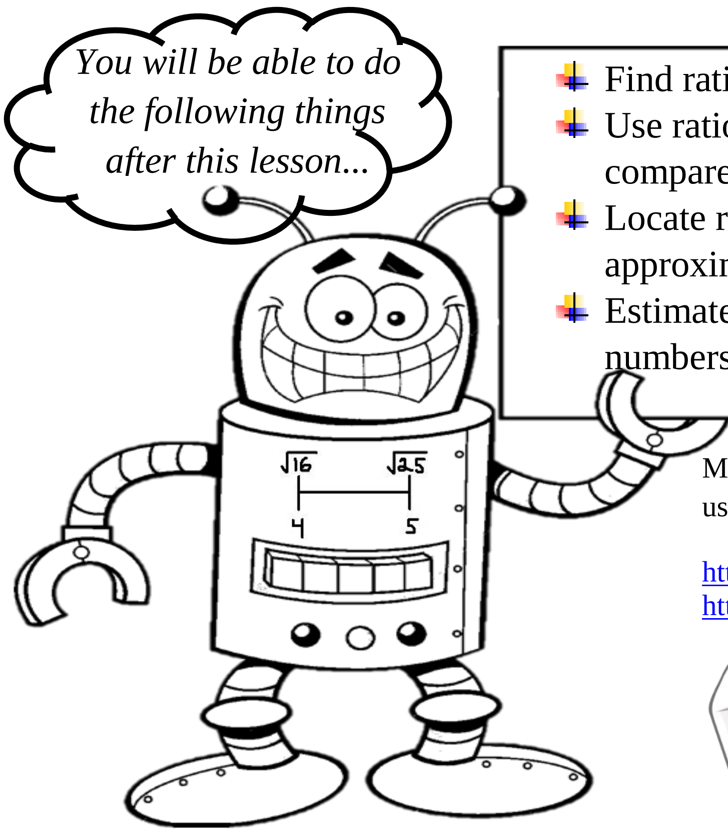
I SCORED BELOW 70% ON THE EDGENUITY

- COMPLETE LESSON **MAFS.8.NS.1.1** ON EDGENUITY
- GO TO LESSON 3: INTEGER EXPONENTS

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 3: RATIONAL AND IRRATIONAL NUMBERS (CONT'D)

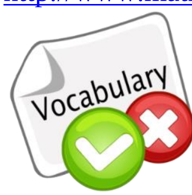
Lesson 3: Rational and Irrational Numbers (Cont'd)



- Find rational approximations of irrational numbers.
- Use rational approximations of irrational numbers to compare the size of irrational numbers.
- Locate rational approximations of irrational numbers approximately on a number line.
- Estimate the value of expressions involving irrational numbers.

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/



- Radical
- Square root
- Perfect square

Evaluating Square Roots of Perfect Squares

Remember that perfect squares are numbers that have integer square roots. Perfect squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256,… and so forth. Since the numbers are perfect squares, we get integer answers when we take the square root. So $\sqrt{1} = 1$ and $\sqrt{36} = 6$ and on and on.

However, strictly speaking each square root has *two solutions*. The reason that $\sqrt{36} = 6$ is because $6 \times 6 = 36$, but notice that $-6 \times -6 = 36$ is also true. That means that $\sqrt{36} = -6$ is a true statement as well. So, for each square root there are two solutions, one positive and one negative.

To prevent confusion about which number we want, the positive or the negative, the mathematical community decided that when we see the square root symbol, we will always give *the principal square root which is the positive answer*. This means that whenever you see something like $\sqrt{49}$ you should know that we only want the positive root, which is 7. If we want the negative root, it will be written like this: $-\sqrt{49} = -7$.

Let's look at a few examples just to make sure we understand.

Check Yourself: Square Roots

1. $\sqrt{144}$ 2. $-\sqrt{100}$ 3. $\sqrt{64}$ 4. $-\sqrt{225}$

It is also possible to ask for both roots. For example, the directions for homework may say, "Find both square roots of the given number." Then you would list them both as follows: $\sqrt{36} = 6$ and -6. The alternate way to list both square roots is to use the plus or minus sign, $\pm$, to represent both. So, you could say $\sqrt{36} = \pm 6$ because the square of 36 is positive 6 and negative 6.

We could also take square roots of certain decimals nicely. For example, $\sqrt{0.36} = 0.6$ or $\sqrt{0.09} = 0.3$.

However, we will limit ourselves to integers for now.

Lastly, remember that we cannot take the square root of negative numbers. So $\sqrt{-64}$ has no solution because nothing times itself is -64. Any number times itself will always be positive.

Evaluating Cube Roots of Perfect Cubes

Just like there are square roots, there are also cube roots. The cube root of a given number is like asking what number cubed (meaning to the third power) will give you that original number. So, the cube root of 8 is 2 because $2^3 = 8$. We represent the cube root with a symbol exactly like the square root symbol except there is a "3" in the "v" of the symbol. Look at the following examples.

$\sqrt[3]{8} = 2$ $\sqrt[3]{27} = 3$ $\sqrt[3]{64} = 4$

The numbers 1, 8, 27, 64, 125, 216, 343, …. and so forth are called perfect cubes because they have an integer cube root.

Notice that a cube root does not have two answers. There are not positive and negative cube roots. Each cube root only has one real number solution. For example, we know $\sqrt[3]{-8} \neq 2$ because $(-2)^3 = -8$ instead of positive eight.

However, this means we can take the cube root of negative numbers. So $\sqrt[3]{-8} = -2$ is a true statement.

Check Yourself: Cube Roots

1. $\sqrt[3]{-1}$
2. $\sqrt[3]{8}$
3. $\sqrt[3]{27}$
4. $\sqrt[3]{-64}$

Approximating Irrational Numbers

It is not always practical to work with irrational numbers. For example, you would not go to the store and order $\sqrt{15}$ packs of bubble gum. Instead it would be better to realize that $\sqrt{15} \approx 4$ and order four packs of bubble gum. How do we make those approximations?

One of the easiest ways to do this is to think of the perfect squares. Recall that the perfect squares are the numbers $1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225,$ and so forth. They are the numbers that have a whole number square root.

If you want to approximate $\sqrt{15}$, notice that $\sqrt{15} \approx \sqrt{16} = 4$. We simply see which perfect square the number inside the square root is closest to and use that to make an estimate. This works well for square roots that are relatively close to a perfect square.

However, some square roots we may want to approximate with more precision. This will mean making an educated guess, checking that guess, and refining that guess. For example, let's approximate $\sqrt{22}$ to one decimal place of accuracy.

First note that $\sqrt{22}$ is between $\sqrt{16}$ and $\sqrt{25}$ which means that our solution is between 4 and 5. The solution is closer to 5 because $\sqrt{22}$ is closer to $\sqrt{25}$. Therefore, we might guess that $\sqrt{22} \approx 4.7$ for an initial guess. Now let's check by examining $4.7 \times 4.7 = 22.09$. That's pretty close, and since $(4.7)^2$ is just over 22, we might check 4.6 to see if it's a better solution. We find that $4.6 \times 4.6 = 21.16$ which is much farther away from 22 than our first guess was. This means that $\sqrt{22} \approx 4.7$ is the most approximate solution to the nearest tenths. Note that if we wanted our solution as an improper fraction, we could easily convert 4.7 to $\frac{47}{10}$.

Let's say that you want to make a square blanket for a baby and want it to be as large as possible. You must buy cloth by the square foot and you can only afford 30 square feet since it costs \$2 per square foot and you have \$60 to spend. What is the approximate side length of the square cloth that you will need to have the fabric store cut for you to the nearest tenths?

$\sqrt{30}$ is between $\sqrt{25} = 5$ and $\sqrt{36} = 6$ and just barely closer to $\sqrt{25}$. So, our first guess might be 5.4. Checking we get that $5.4 \times 5.4 = 29.16$, which is a little below the 30 we are looking for. So now we try.

$5.5 \times 5.5 = 30.25$, which is just above 30, but is a closer estimate. Therefore, we will say $\sqrt{30} \approx 5.5$ and have the fabric store cut a cloth square that is 5.5 feet by 5.5 feet.

Check Yourself: Approximating irrational numbers to the nearest whole number

1. $\sqrt{2}$ 2. $\sqrt{41}$ 3. $\sqrt{77}$ 4. $\sqrt{120}$

Using Technology

Calculators give you an approximation of irrational numbers whenever you find a square root of a non- perfect square. For example, plugging in $\sqrt{22}$ to a calculator gives us $\sqrt{22} \approx 4.6904157598$. This is an approximation because we know that the actual solution as a decimal goes on forever, but the calculator has to stop at some point and display the answer. If you were asked to *round* the solution of $\sqrt{22}$ to one decimal place, then you could simply plug in $\sqrt{22}$ to your calculator and then round it to $\sqrt{22} \approx 4.7$.

Most calculators also have a square root button that looks like $\boxed{\sqrt{}}$ or $\boxed{\sqrt{x}}$. They also have another root button that is used for cube roots or even higher roots that looks like $\boxed{\sqrt[x]{y}}$. To use this button, type the number you want the root of, then hit the button, then which root you want. So, if you wanted to find $\sqrt[3]{-8}$ you would type in -8, hit the $\boxed{\sqrt[x]{y}}$ button, then type 3 before hitting equals. While the calculator can perform these operations, it will be unnecessary since we confine ourselves to small perfect squares and cubes. In other words, you should be working out these types of problems by hand.

Comparing and Ordering Irrational Numbers on a Number Line

To compare irrational numbers that are square roots, we can simply examine the number that we are taking the square root of. For example, we know that $\sqrt{15} < \sqrt{17}$ because 15 is less than 17.

However, when we compare irrational numbers such as $\sqrt{10}$ and π, it is easier to compare rational approximations of each written as a decimal. We know that $\sqrt{10} \approx 3.16$ and that $\pi \approx 3.14$. Therefore, we can say that $\sqrt{10} > \pi$. Notice that it was useful to approximate the irrational numbers to two decimal places, hundredths place.

The same is true for comparing irrational and rational numbers. By finding a rational approximation of the irrational numbers, we can compare values such as π and $\frac{22}{7}$. For these numbers we may have to use a number in the thousandths for our approximation, the use of a calculator would make sense. Rounded to the nearest thousandths, we find that $\pi \approx 3.142$ and $\frac{22}{7} \approx 3.143$ which means that $\pi < \frac{22}{7}$.

Once we know how to compare two numbers, we can then order a set of numbers through comparison of two numbers at a time. For example, we could list from least to greatest $\sqrt{10}, \pi, 3.14,$ and $\frac{22}{7}$. We know the following:

$$\sqrt{10} \approx 3.162$$
$$\pi \approx 3.142$$
$$\frac{22}{7} \approx 3.143$$

We see that $\sqrt{10}$ is greater than all the other numbers given. We also note that 3.14 is the smallest because it is equal to 3.140 to three decimal places, thousandths. Therefore, we can list them in order like so: $3.14, \pi, \frac{22}{7}, \sqrt{10}$. Notice that the closer the numbers are to each other, the more decimal places of accuracy we need in our rational approximation.

Check Yourself: Compare the following numbers using < or >.

1. $\sqrt{51}$ $\square$ 5.1

2. $\sqrt{38}$ $\square$ $\sqrt{42}$

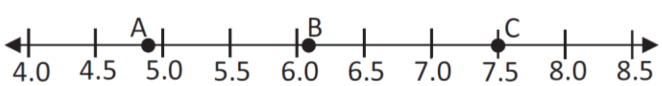
3. $\sqrt{17}$ $\square$ $\frac{9}{2}$

4. $\sqrt{49}$ $\square$ 7.1

Locating Irrational Numbers on a Number Line

Again, rational approximations of irrational numbers will be our friend. On a number line, we generally list rational number markers. On the simplest number lines, we count by integers. On a standard English ruler, we count by fractions, usually $\frac{1}{16}$ $inch$ or $\frac{1}{8}$ $inch$. On a standard metric ruler, we count by millimeters which are each 0.1 centimeters. No matter how the number line is set up, we will still need the rational approximations of the irrational numbers.

For example, let's try to place the following irrational numbers on the number line: $\sqrt{37}$, $\sqrt{42}$, and $\sqrt{24}$. First, we will make a quick, one decimal place (tenths) approximation of each.$\sqrt{37} \approx 6.1$ since 37 is just over 36, $\sqrt{56} \approx 7.5$ since 56 is about half-way between the perfect squares of 49 and 64,and $\sqrt{24} = 4.9$ since 24 is just under 25. Now examine where the dots are located on the following number line.

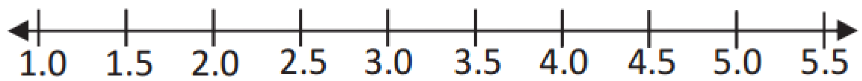


Note that point A must be $\sqrt{24}$ since it is just under 5, point B must be $\sqrt{37}$ since it is just over 6, and point C must be $\sqrt{56}$ since it is right at 7.5 on the number line.

In the same way that you can identify which point on a number line goes with which irrational number, you can also place points on a number line to represent the irrational number.

Check Yourself: Put a point on the line for each irrational number

1. Point A: $\sqrt{2}$
2. Point B: $\sqrt{17}$
3. Point C: $\sqrt{11}$
4. Point D: $\sqrt{5}$



Estimating Values of Expressions

Surprise! We're going to use rational approximations of irrational numbers again. We're basically going to be looking at adding, subtracting, and multiplying irrational numbers and how to quickly estimate an answer.

For example, let's try to add the following irrational numbers: $\sqrt{37} + \sqrt{24}$. As a very fast estimate, we know that $\sqrt{37} \approx 6$ since 37 is just over 36 and $\sqrt{24} \approx 5$ since 24 is just under 25. That means we would estimate $\sqrt{37} + \sqrt{24} \approx 11$.

We could further fine tune our estimates by approximating the irrational numbers to one decimal place, tenths. Here's an example:

$$\sqrt{37} + \sqrt{56} \approx 6.1 + 7.5 \approx 13.6$$

One last concept we need to be familiar with is multiplication in solving irrational numbers. Recall that the expression $5x$ means five times$x$. In the same way, $5\sqrt{15}$ means five times the square root of fifteen. To estimate that expression we can approximate in the following way:

$$5\sqrt{15} \approx 5(4) \approx 20 \text{ or with more precision } 5\sqrt{15} \approx 5(3.9) \approx 19.5$$

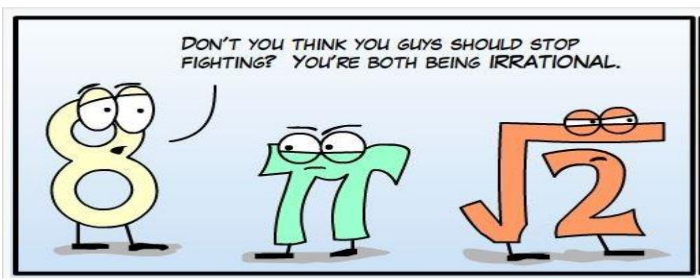
Now combining all those qualities we can estimate more complex expressions involving all of the operations of addition, subtraction, and multiplication. Just don't forget to follow the order of operations! For example, to the nearest whole number we could estimate the following expression:

$$2\sqrt{13} + 5\sqrt{5} - \sqrt{37} \approx 2(3.5) + 5(2) - 6 \approx 11$$

Note that it was useful to approximate $\sqrt{13}$ to 3.5 in the middle of the problem since it led to a nice whole number solution at the end. In general, if we want a whole number answer, it might be a good idea to approximate each irrational as either a whole number or the nearest half value.

Check Yourself: Estimate the expressions to the nearest whole number

1. $\sqrt{8}+\sqrt{18}$
2. $11-\sqrt{80}$
3. $4\sqrt{15}-5$
4. $4\sqrt{24}-3\sqrt{3}$



DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- COMPARE IRRATIONAL AND RATIONAL NUMBERS
- UNDERSTAND AND APPLY THE DEFINITION OF IRRATIONAL NUMBERS

TAKE THE **MAFS.8.NS.1.2** EDGENUITY PRE-TEST

YES

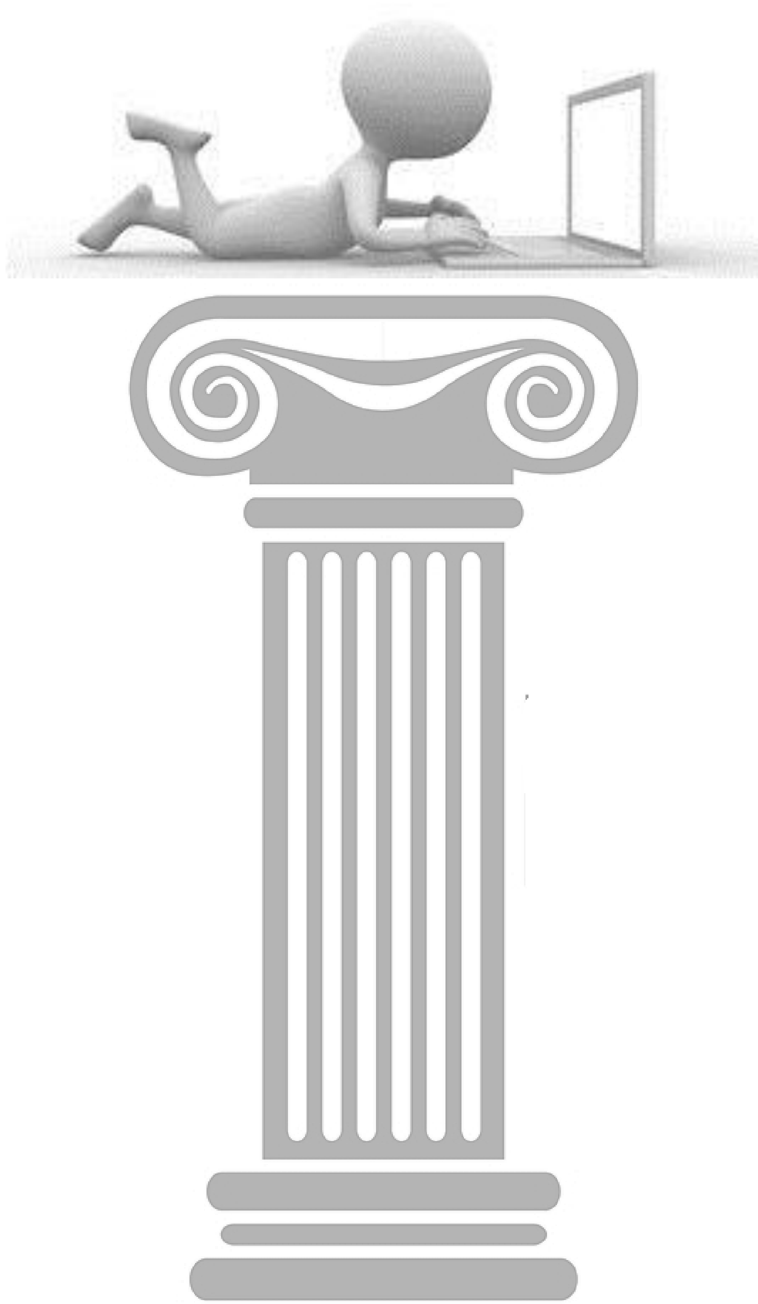
GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.NS.1.2**

I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

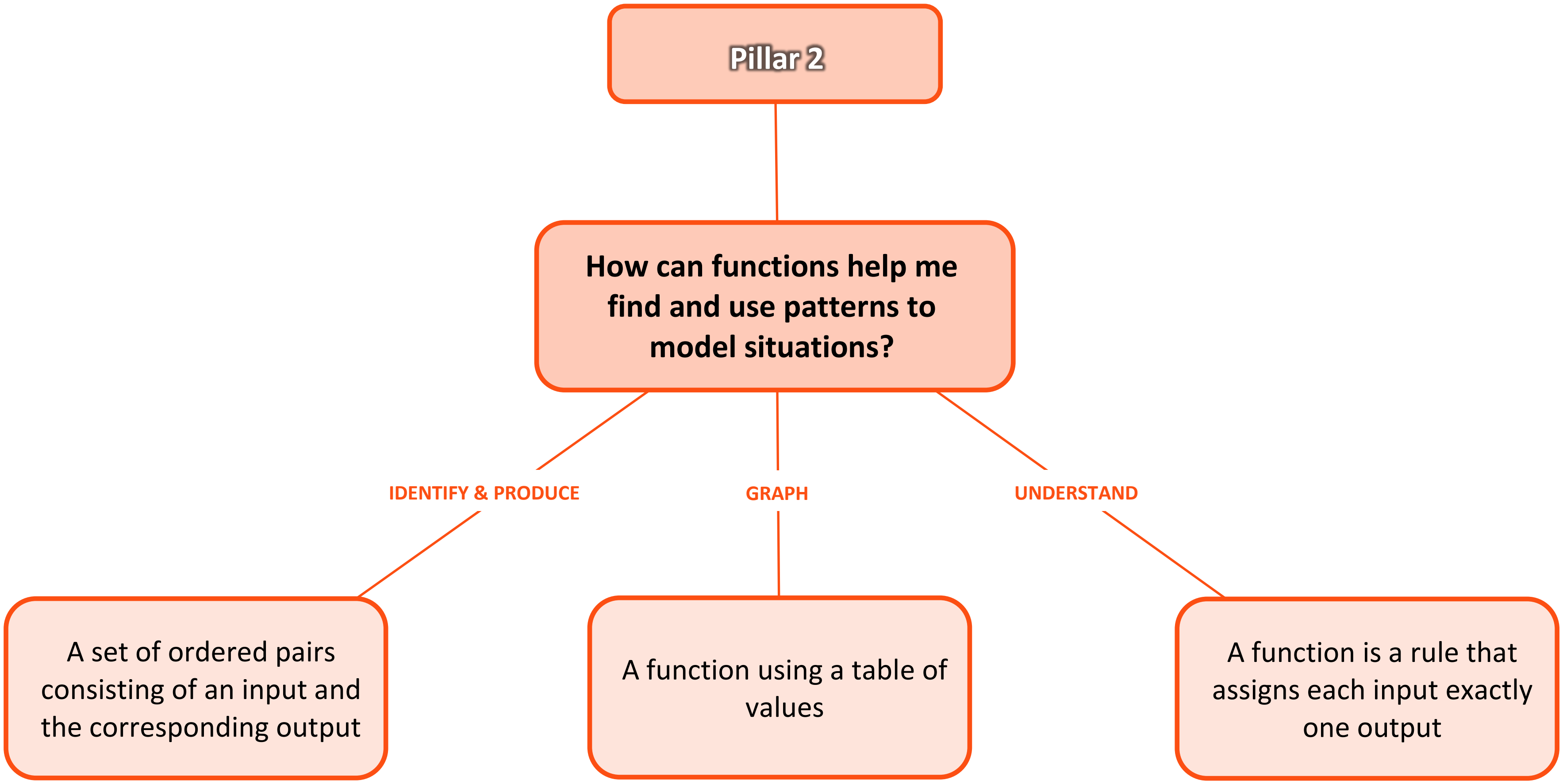
- COMPLETE LESSON **MAFS.8.NS.1.2** ON EDGENUITY
- GO TO LESSON 4: INTRODUCTION TO FUNCTIONS

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 4: INTRODUCTION TO FUNCTIONS

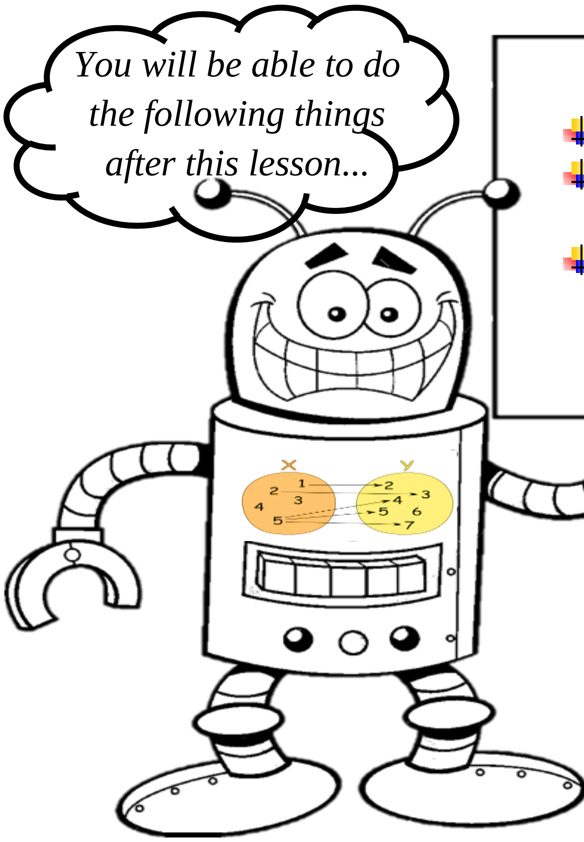


PILLAR #2



Algebra 1: Functions » Interpreting Functions

Lesson 4: Introduction to Functions



- Determine if an equation represents a function.
- Apply a function rule for any input that produces exactly one output.
- Generate a set of ordered pairs from a function and graph the function.

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

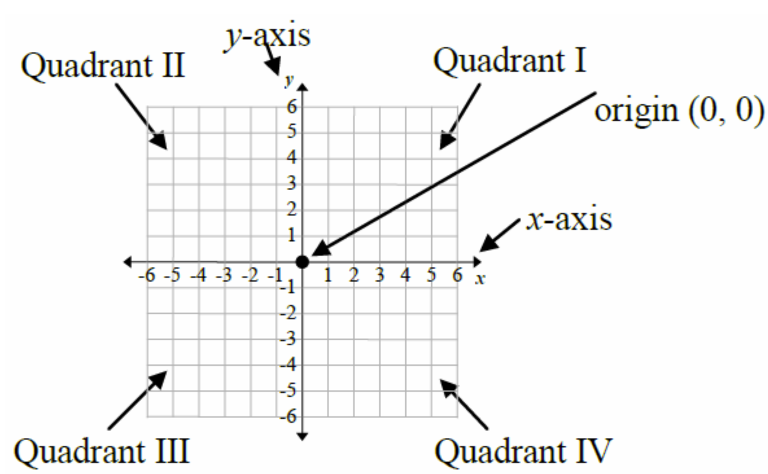
- Ordered pairs
- Coordinate plane
- X-coordinate
- Y-coordinate
- Input
- Output
- Domain
- Range
- Relation
- Function
- Linear line test

Review - The Coordinate Plane

A coordinate plane is formed by the intersection of a horizontal number line called the *x-axis* and vertical number line called the *y-axis***.**

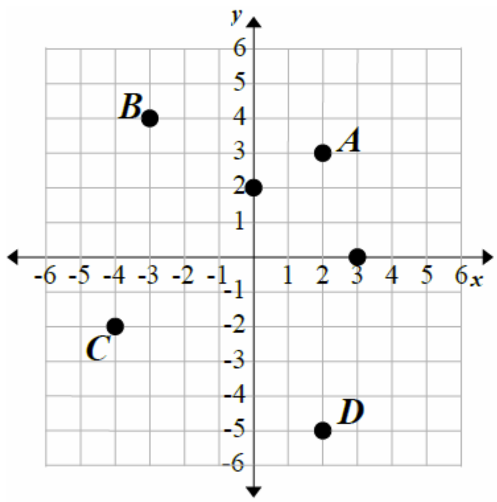
The *x*-axis and *y*-axis meet or intersect at a point called the *origin*..

The coordinate plane has seven key features: the *x*-axis, the *y*-axis, the origin, Quadrant I, Quadrant II, Quadrant III, and Quadrant IV. Refer to the diagram on the right.



Hint: one way to remember the order of the quadrants is to think of writing a “C” (for coordinate plane) around the origin. To create the “C” you start in quadrant I and move counterclockwise (and so does the numbering of the quadrants).

The coordinate plane consists of infinitely many points called *ordered pairs*. Each ordered pair is written in the form of (x, y). The first coordinate of the ordered pair corresponds to a value on the x-axis and the second number of the ordered pair corresponds to a value on the y-axis. Our movements in the coordinate plane are similar to movements on the number line. As you move from left to right on the x-axis, the numbers are increasing in value. The numbers are increasing in value on the y-axis as you go up.



To find the coordinates of point ***A*** in Quadrant I, start from the origin and move 2 units to the right, and up 3 units. Point ***A*** in Quadrant I has coordinates of $(2, 3)$.

To find the coordinates of point ***B*** in Quadrant II, start from the origin and move 3 units to the left, and up 4 units. Point ***B*** in Quadrant II has coordinates of $(-3, 4)$.

To find the coordinates of the point ***C*** in Quadrant III, start from the origin and move 4 units to the left, and down 2 units. Point ***C*** in Quadrant III has coordinates of$(-4, -2)$.

To find the coordinates of the point ***D*** in Quadrant IV, start from the origin and move 2 units to the right, and down 5 units. Point ***D*** in Quadrant IV has coordinates of $(2, -5)$.

Here are a few phrases that teachers have used with students to help them remember how to determine the coordinates of a point. “Run before you jump” implies moving right or left before you move up or down.

Notice that there are two other points on the above graph, one point on the x-axis and the other on the y-axis. For the point on the x-axis, you move 3 units to the right and do not move up or down. This point has coordinates of (3, 0). For the point on the y-axis, you do not move left or right, but you do move up 2 units on the y-axis. This point has coordinates of (0, 2). Points on the x-axis will have coordinates of $(x, 0)$ and points on the y-axis will have coordinates of $(0, y)$. The *first* number in an ordered pair tells you *to move left or right* along the x-axis. The *second* number in the ordered pair tells you *to move up or down* along the y- axis.

Relations and Functions

Students that have read a menu have experienced working with ordered pairs. Menus are typically written with a food item on the left side of the menu, the cost of the item on the other side as shown:

Hamburger	**$3.50**
Pizza	**$2.00**
Sandwich	**$4.00**

Menus could have just as well been written horizontally:

$$\boldsymbol{Hamburger}, \$\mathbf{3.50}, \boldsymbol{Pizza}, \mathbf{2.00}, \boldsymbol{Sandwich}, \mathbf{4.00}$$

But that format (notation) is not as easy to read and could cause confusion. Someone might look at that and think you could buy a $2.00 sandwich. To clarify that, I might group the food item and its cost by putting parentheses around them:

$$(\boldsymbol{Hamburger}, \$\mathbf{3.50}), \quad (\boldsymbol{Pizza}, \mathbf{2.00}), \quad (\boldsymbol{Sandwich}, \mathbf{4.00})$$

Those groupings would be called ordered pairs because there are two items. Ordered because food is listed first, cost is second.

By definition, a *relation is any set of ordered pairs*.

Another example of a set of ordered pairs could be described when buying cold drinks. If one cold drink cost $0.50, two drinks would be $1.00, and three drinks would be $1.50. I could write those as ordered pairs:

$(1, .50)$, $(2, 1.00)$, $(3, 1.50)$, and so on.

From this pattern, you would expect the cost to increase by $0.50 for each additional drink. What do you think might happen if one student went to the store and bought 4 drinks for $2.00 and his friend who was right behind him at the counter bought 4 drinks and only paid $1.75?

My guess is the first guy would feel cheated, that it was not correct, that this was not working, or that this was not "functioning". The first guy would expect anyone buying four drinks would pay

$2.00, just like he did.

Let's look at the ordered pairs that caused this problem.

$$(1, .50), \quad (2, 1.00), \quad (3, 1.50), \quad (4, 2.00), \quad (4, 1.75)$$

The last two ordered pairs highlight the malfunction, one person buying 4 drinks for $2.00, the next person buying 4 drinks for a $1.75. For this to be fair or functioning correctly, we would expect that anyone buying four drinks would be charged $2.00. Or more generally, we would expect every person who bought the same number of drinks to be charged the same price. When that occurs, we'd think this is functioning correctly. So, let's define a function.

*A **function** is a special relation in which no two ordered pairs have the same first element or x-value.*

Since the last set of ordered pairs has the same first elements (x-value), those ordered pairs would not be classified as a function.

If I asked students how much 10 cold drinks would cost, many might realize the cost would be \$5.00.

If I asked them how they got that answer, someone might tell me they multiplied the number of cold drinks by \$0.50. That shortcut can be described by a rule:

$cost\ =\ \$0.50\ \times\ number\ of\ cold\ drinks$

$c = 0.50n$

or you may see it written as:

$y = 0.50x$ or $y = \frac{1}{2}x$

That rule generates more ordered pairs. So, if I wanted to know the price of 20 cold drinks, I would substitute 20 for x. The result would be \$10.00. Written as an ordered pair, I would have $(20, 10)$.

Let's look at another rule.

Example: $y = 3x + 2$

If we plug 4 in, we will get 14 out, represented by the ordered pair $(4, 14)$.

If we plug 0 in, we get 2 out, represented by $(0, 2)$.

There is an infinite number of numbers I can plug in.

A *relation* is any set of ordered pairs. The *set of all first numbers of the ordered pairs (inputs)* is called the *domain* of the relation. The set of *second numbers (outputs)* is called the *range* of the relation.

Sometimes we put restrictions on the numbers we plug into a rule (the domain). Those restrictions may be placed on the relation so it fits real world situations.

For example, let's use our cold drink problem. If each drink costs $\$0.50$, it would not make sense to find the cost of -2 drinks. You can't buy negative two drinks, so we would put a restriction on the domain. The only numbers we could plug in are whole numbers: $0, 1, 2, 3$ ...

The restriction on the domain also affects the range. If you can only use positive whole numbers for the domain, what values are possible for the range?

We defined a *function* as a special relation in which every domain, the first number in an ordered pair, produces only one unique range value.

What this means is that for every member of the domain, there is one and only one member of the range. If I give you a rule, like $y = 2x - 3$, when I plug in4 for the x-value, I get 5 as my output. This is represented by the ordered pair $(4, 5)$. Now if I plug in 4 again, I have to get 5 as my output. That makes sense—it's expected—so just like the rule for buying cold drinks, this is working, this is functioning as expected, it's a function.

Now, you are thinking, big deal, isn't that what we would expect?

Looking at another rule might give us a clue, $x^2 + y^2 = 25$

Solving that for *y*, we get

$$y^2 = 25 - x^2$$

$$y = \pm\sqrt{25 - x^2}$$

Now if we plug in a number like 3, we get two answers: $(3, 4)$ and $(3, -4)$. You can see there is not one and only one member in the range for each member in the domain. Therefore, this rule describes a relation that is *not* a function.

We can look at the graphs of relations that are nothing more than a bunch (set) of ordered pairs (points) and determine if it's a function.

To determine if a graph describes a function, you use what we call the <u>*vertical line test*</u>. That is, you try to draw a vertical line through the graph so it intersects the graph in more than one point. If you can do that, then those two ordered pairs have the same first element, but a different second element. Therefore, the graph would *not* describe a function.

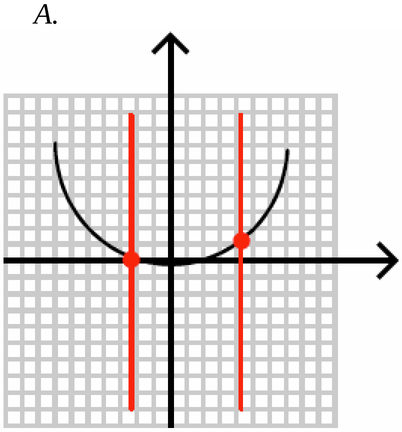
If there does not exist any vertical line which crosses the graph of the relation in more than one place, then the relation is a function. This is called the VERTICAL LINE TEST.

What we try to do is draw a vertical line so it intersects the graph in more than one place. If we can't, then we have a graph of a function.

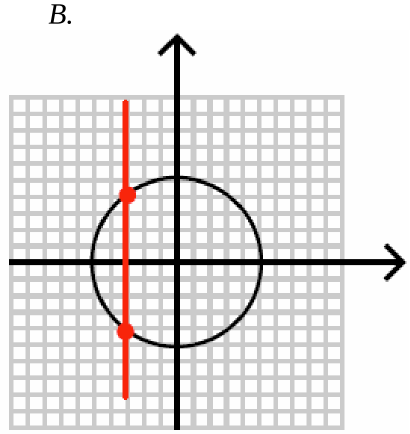
Let's try a few problems.

Label the following as relations or functions.

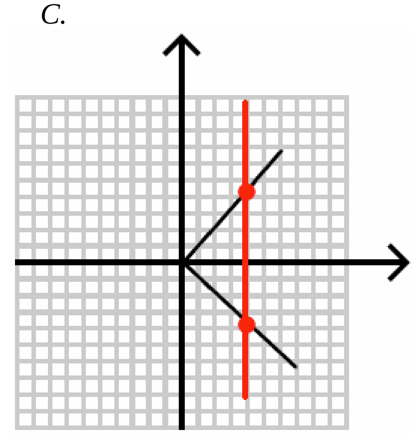
A.



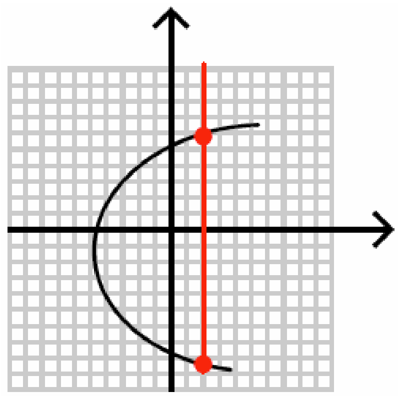
B.



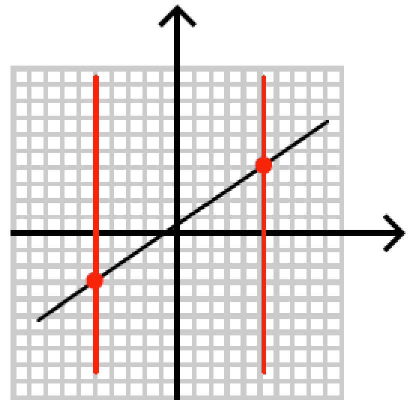
C.



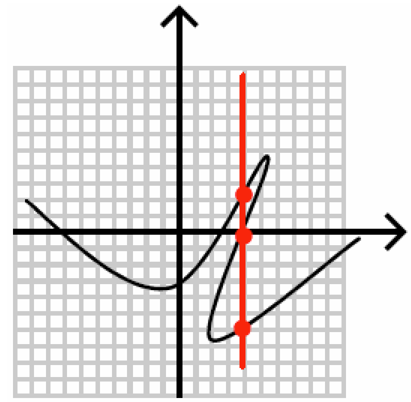
D.



E.



F.



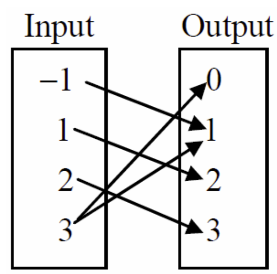
Only *A* and *E* are functions.

In addition to looking at a graph to represent a relation, you can use a *mapping diagram*.

Example: Represent the relation (-1, 1), (3, 0), (3, 1), (2, 3), (1, 2) *using a mapping diagram.*

List the inputs and the outputs in order.

Draw arrows from the inputs to their outputs.

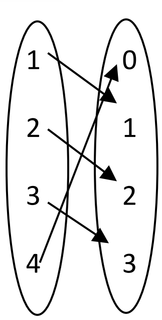


Is this relation a function? No, because the input 3 is paired with two outputs, 0 and 1. You could also verify this by graphing the ordered pairs and applying the vertical line test.

Check Yourself: Convert the following decimals to fractions

1. Which relation is NOT a function?

A.



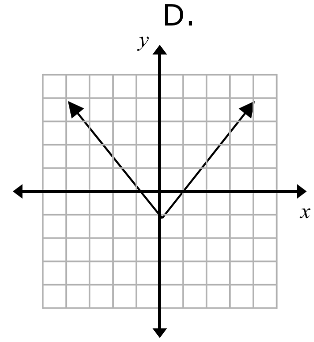
B.

x	1	2	3
y	6	6	6

C.

$$\{(1, 2), (1, 3), (1, 4)\}$$

D.



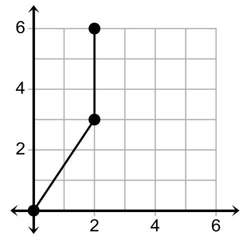
2. Select **all** relations that are functions.

A. $y=\frac{1}{2}x+\frac{1}{3}$

B.

x	y
-3	-7
-1	-1
-1	-4
0	2
4	10

C.



D.

$\{(-1,\ 0)\ (0,\ 1)\ (1,\ 1)\ (2,\ 2)\}$

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- UNDERSTAND A FUNCTION AS A TYPE OF RELATION
- DEFINE A FUNCTION BY LOOKING AT ITS PARTS
- DETERMINE WHETHER A SET OF POINTS PLOTTED ON A GRAPH IS A FUNCTION
- DETERMINE WHETHER A GRAPH IS A FUNCTION
- DETERMINE WHETHER A SET OF ORDERED PAIRS REPRESENTS A FUNCTION

TAKE THE **MAFS.8.F.1.1** EDGENUITY PRE-TEST

YES

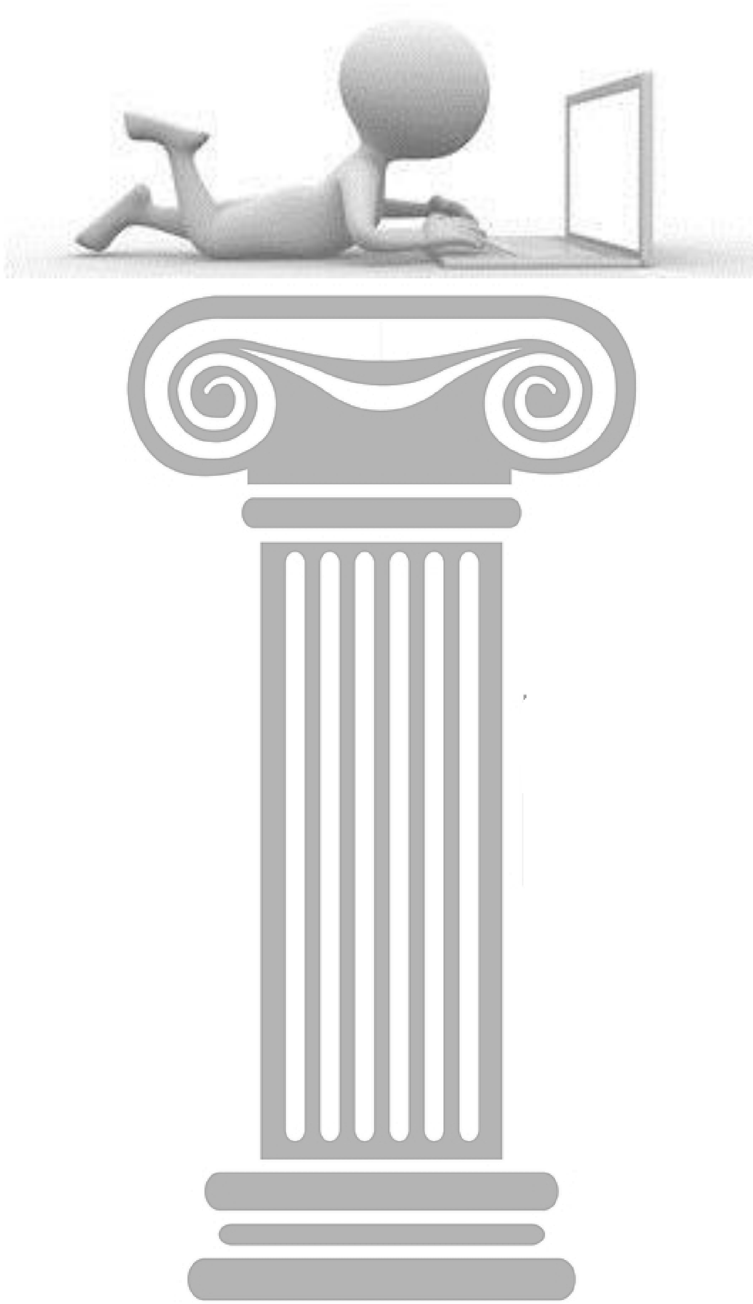
GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.F.1.1**

I SCORED BELOW 70% ON THE EDGENUITY

- COMPLETE LESSON **MAFS.8.F.1.1** ON EDGENUITY
- GO TO LESSON 5: PROPORTIONAL RELATIONSHIP

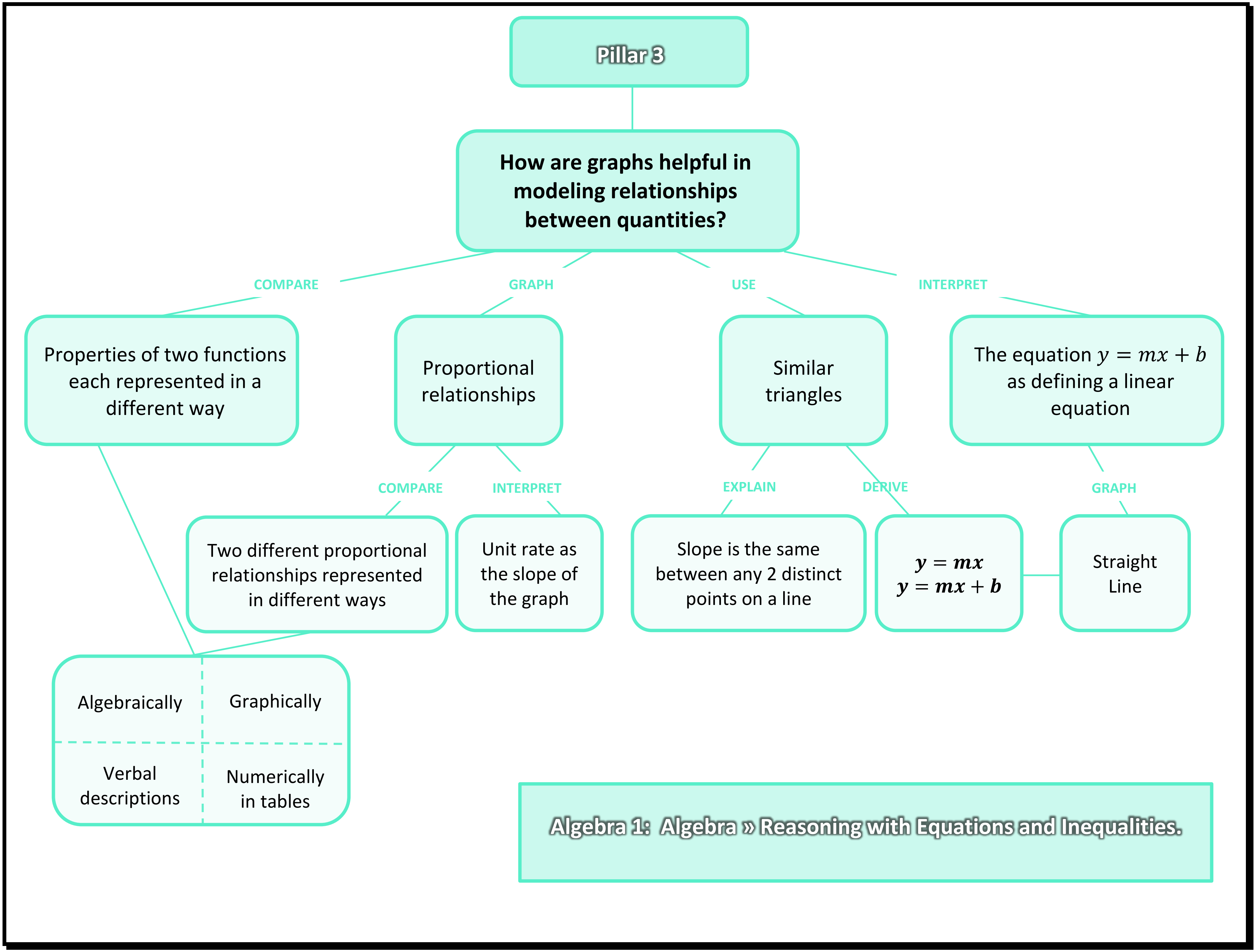
I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 5: PROPORTIONAL RELATIONSHIP



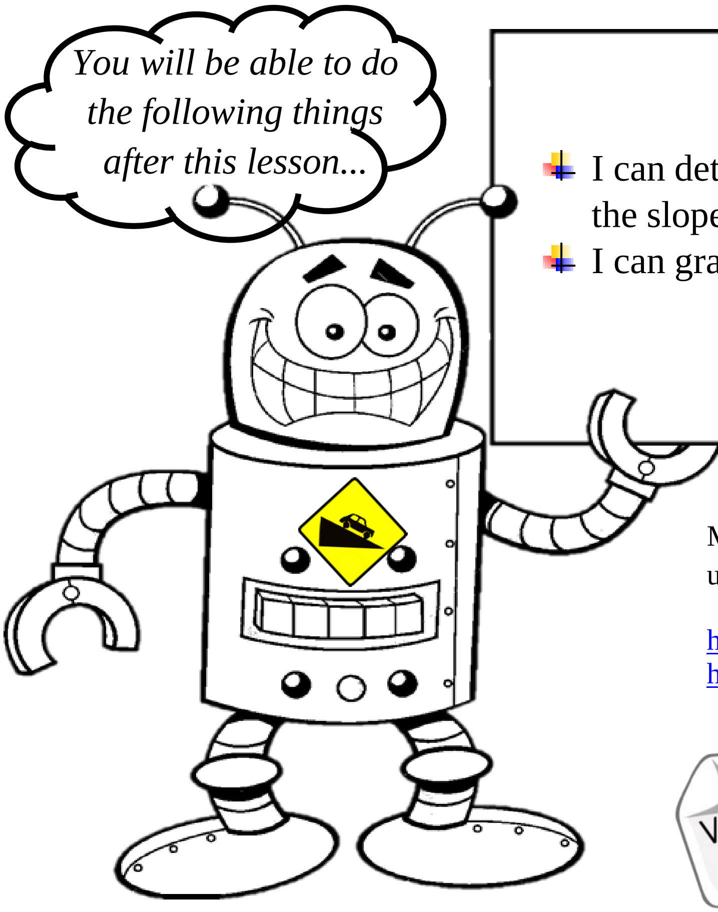
PILLAR #3

The resources in this packet have been compiled from the Regional Professional Development Program and Mr. Bright



Pillar 3
How are graphs helpful in modeling relationships between quantities?
COMPARE
GRAPH
USE
INTERPRET
Properties of two functions each represented in a different way
Proportional relationships
Similar triangles
The equation $y = mx + b$ as defining a linear equation
COMPARE
INTERPRET
EXPLAIN
DERIVE
GRAPH
Two different proportional relationships represented in different ways
Unit rate as the slope of the graph
Slope is the same between any 2 distinct points on a line
$\boldsymbol{y = mx}$
$\boldsymbol{y = mx + b}$
Straight Line
Algebraically
Graphically
Verbal descriptions
Numerically in tables
Algebra 1: Algebra » Reasoning with Equations and Inequalities.

Lesson 5: Proportional Relationships



- I can determine the rate of change by the definition of the slope.
- I can graph proportional relationships.

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- Constant of Proportionality
- Direct Variation
- Proportional Relationship
- Rate Of Change
- Rise/Run
- Slope
- Unit Rate

Slope

The idea of slope is used quite often in our lives. However, outside of school, it goes by different names. People involved in home construction might talk about the pitch of a roof. If you were riding in your car in a hilly area, you might have seen a sign on the road indicating a grade of 6% up or down a hill. Both of those cases refer to what we call slope in mathematics.

Kids use slope on a regular basis without realizing it. Let's look at our drink example again. A student buys a cold drink for \$0.50. If two cold drinks were purchased, the student would have to pay \$1.00. I could describe that mathematically by using ordered pairs: (1, \$0.50), (2, \$1.00), (3, \$1.50), and so on. The first element in the ordered pair represents the number of cold drinks; the second number represents the cost of those drinks. Easy enough, don't you think?

Now if I asked the student how much more was charged for each additional cold drink, hopefully the student would answer \$0.50. So, the difference in cost from one cold drink to adding another is \$0.50. The cost would change by \$0.50 for each additional cold drink. The change in price for each additional cold drink is \$0.50. Another way to say that is the *rate of change* is \$0.50. We call the rate of change *slope*.

In math, the *rate of change* is called the *slope* and is often described by the ratio $\frac{rise}{run}$.

The rise represents the change (difference) in the vertical values (the *y*'s); the run represents the change in the horizontal values, (the *x*'s). Mathematically, we write

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Let's look at any two of those ordered pairs from buying cold drinks: (1, \$0.50) and (3, \$1.50). Find the slope. Substituting in the formula, we have:

$$\begin{aligned} m &= \frac{1.50 - 0.50}{3 - 1} \\ &= \frac{1.00}{2} \\ &= 0.50 \end{aligned}$$

We find the slope is \$0.50. The rate of change per drink is \$0.50.

Example: *Find the slope of the line that connects the ordered pairs* $(3, 5)$ *and* $(7, 12)$.

To find the slope, I use $m = \frac{y_2 - y_1}{x_2 - x_1}$

Subtract the *y* values and place that result over the difference in the *x* values.

$\frac{12-5}{7-3} = \frac{7}{4}$ The slope is $\frac{7}{4}$

*Example***:** *Find the slope of the line on the graph to the right.*

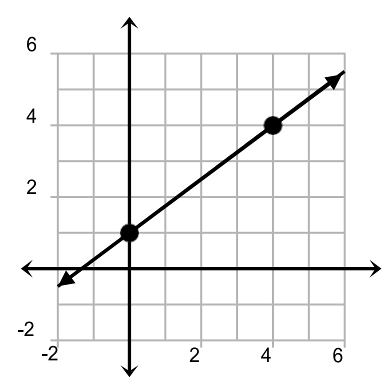
Pick two points that are easy to identify. $(0, 1)$ and $(4, 4)$

Find the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4 - 1}{4 - 0} = \frac{3}{4}$$

The slope of the graphed line is $\frac{3}{4}$



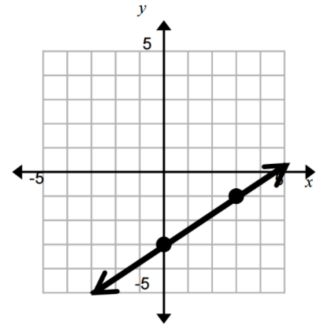
Check Yourself: Find the slope of the line through the given points.

1. $(5, 4), (2, 0)$

2. $(-3, 6), (4, 6)$

3. $(3, -1), (3, 2)$

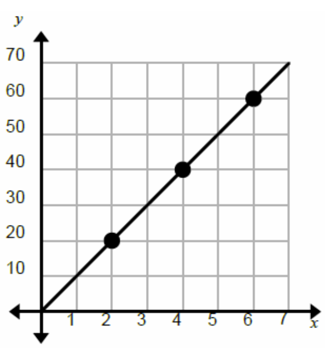
4.



Direct Variation: Constant of Proportionality (Variation), Slope & Unit Rate

Consider the table. Note that the ratio of the two quantities is constant $\left(\frac{20}{2} = \frac{40}{4} = \frac{60}{6}\right)$ indicating a proportional relationship. This relationship is called a *direct variation*. This constant ratio is called the *constant of proportionality* or *constant of variation.*

Babysitting (hours), ***x***	*Money Earned ($),* ***y***
2	20
4	40
6	60



Consider the graph of a line containing these points.

Determine the slope: $\frac{change\ in\ y}{change\ in\ x} = \frac{20}{2} = 10$

What does this mean? *$10 is earned per hour babysitting*

Recall, this is also called the unit rate (a rate with 1 in the denominator).

Therefore, note that the *constant of proportionality (variation)*, *the slope, and the unit rate all have the same value.*

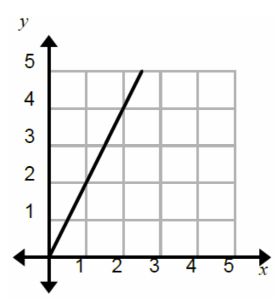
Check Yourself: Determine the unit rates

1. Bamboo that grows 5 inches in 2.5 hours.

2.

Cyclist Ride	
Hours	Miles
3	24
6	48

3.



Representing Proportional Relationships and Slope

A *proportional relationship* between two quantities exists if they have a constant ratio and a constant rate of change. This relationship is also called a *direct variation*. The equations of such relationships are always in the form $y = mx$. When graphed, they produce a line that passes through the origin. In this equation, m is the *slope* of the line; it is also called the *unit rate*, the rate of change, or the *constant of proportionality* of the function.

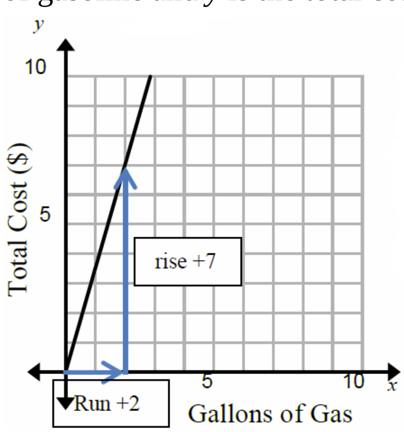
Example: *Graph the proportional relationship between the two quantities, write the equation representing the relationship, and describe how the unit rate or slope is represented on the graph.*

Gasoline cost \$3.50 per gallon

We can start by creating a table to show how these two quantities, gallons of gas and cost, vary. Two things show us that this is a proportional relationship. First, it contains the origin, (0, 0), and this makes sense: if we buy zero gallons of gas it will cost zero dollars. Second, if the number of gallons is doubled, the cost is doubled; if it is tripled, the cost is tripled.

Gas (gal)	Cost (\$)
0	0
1	3.50
2	7
3	10.50

The equation that will represent this data is $y = 3.50x$, where x is the number of gallons of gasoline and y is the total cost ($y = mx$). Slope is 3.50 as indicated in the table as the unit.



The graph is shown. (Note: The equation does extend into the third quadrant because this region does not make sense for the situation. We will not buy negative quantities of gasoline, nor pay for it with negative dollars!).

We can find the slope by creating a "slope triangle" which represents $\frac{rise}{run} = \frac{7}{2} = 3.5$, which confirms the slope we show in the equation.

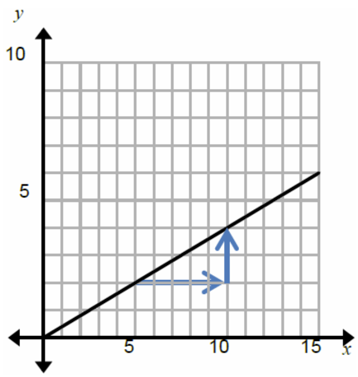
Either way, the constant of proportionality is the slope, which is 3.5 gallons of gas per cost.

Example: *Graph the proportional relationship between the two quantities, write the equation representing the relationship, and describe how the unit rate or slope is represented on the graph.*

Five Fuji Apples cost \$2

Again, we can begin by creating a table relating the number of apples to their cost. We can use this table to plot the points and determine the slope of the line.

# of apples	0	5	10	15
cost ($)	0	2	4	6



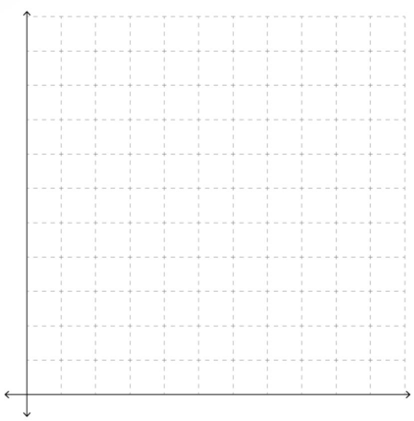
Using the slope triangle, we can see that $\frac{rise}{run} = \frac{2}{5}$.

Using $y = mx$, the equation for the line is $y = \frac{2}{5}x$.

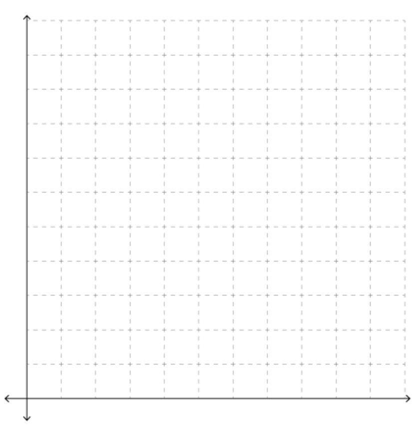
For unit rate: if five apples cost $2.00, then one apple costs $\frac{2.00}{5} =$ 0.40 or 40 cents per apple. (It is also represented on the graph: for one apple, the graph rises 0.40.)

Check Yourself: Graph the proportional relationship between the two quantities, write the equation representing the relationship, and describe how the unit rate or slope is represented on the graph.

1. In a food eating contest, a contestant eats 60 hot dogs in 10 minutes.



2. Every six days, Britney receives 2 emails.



Comparing Proportional Relationships in Different Formats

You can use table, graphs, words or equations to represent and compare proportional relationships. Different cyclists' rates are represented below.

Words Cyclist *A*

A cyclist can ride 24 miles in 2 hours.

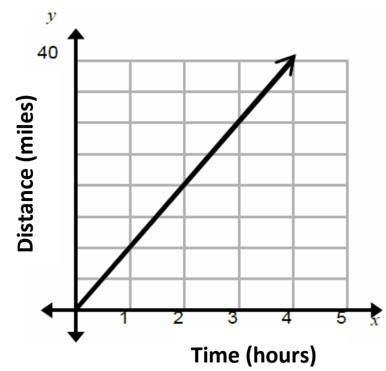
Equation Cyclist *C*

$$y = 9x$$

Table Cyclist *B*

Time Hours	Distance (miles)
0	0
1	5
2	10
3	15

Graph Cyclist *D*



Which cyclist is faster? *Cyclist A* is faster. Which cyclist is slower? *Cyclist* B is slower. Explain your reasoning. *Cyclist A's rate is* $\frac{24\ miles}{2\ hours} = \frac{12\ miles}{1\ hour}$, *cyclist B's rate is 5mph, cyclist C's rate is 9mph and Cyclist D's rate is 10mph.*

Check Yourself: Compare proportional relationships

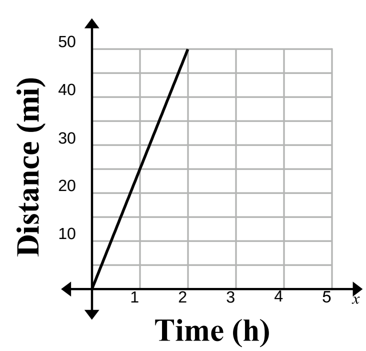
1. Earnings for 3 weeks from a part time job are shown in the table.

Time Worked (h)	11	21	8
Total Pay ($)	78.65	150.15	57.20

   He is offered a job that will pay $7.25 per hour. Which job pays better? Explain your reasoning.

2. The distance *d* in miles that can be covered by a rabbit in *t* hours is given in the equation $d = 30t$.
   The distance covered by a bear is shown on the graph.

   Which animal is faster? Explain your reasoning.



DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- DETERMINING THE CONSTANT RATE OF CHANGE
- DISPLAY ALL POSSIBILITIES IN A PROPORTIONAL RELATIONSHIP BY GRAPHING
- FIND A UNIT RATE USING A GRAPH

TAKE THE **MAFS.8.EE.2.5** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.EE.2.5**

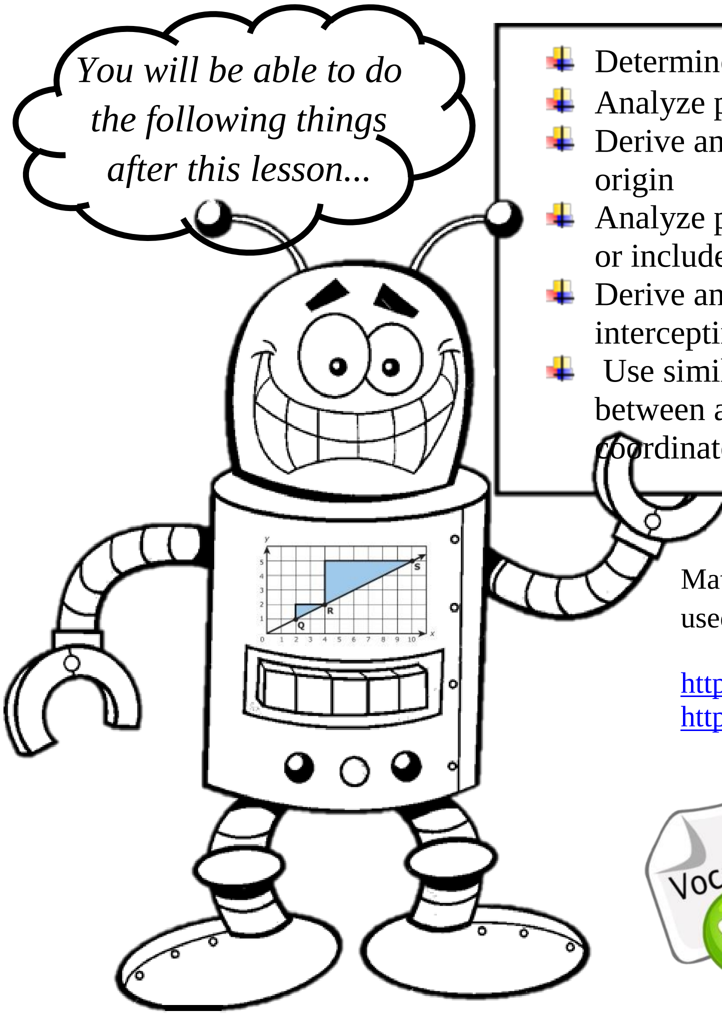
I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

- COMPLETE LESSON **MAFS.8.EE.2.5** ON EDGENUITY
- GO TO LESSON 6: LINEAR EQUATIONS IN ONE VARIABLE

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 6: LINEAR FUNCTIONS

Lesson 6: Linear Functions



- Determine the y-intercept of a line.
- Analyze patterns for points on a line through the origin
- Derive an equation of the form y= mx for a line through the origin
- Analyze patterns for points on a line that does not pass through or include the origin
- Derive an equation of the form y= mx + b for a line intercepting the vertical axis at b (the y-intercept)
- Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane

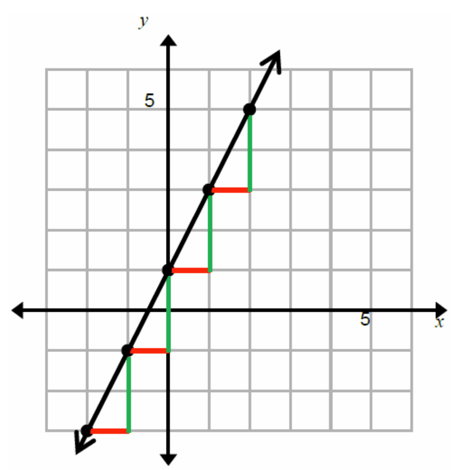
Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

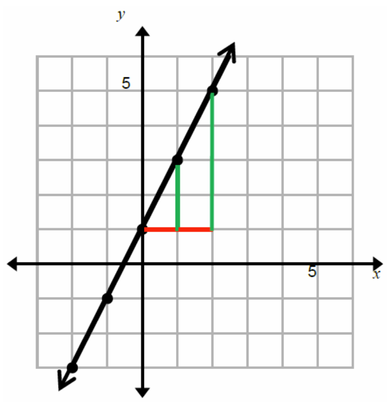
https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- Similar Triangles

Using Similar Triangles to Explain Slope

Look at the line graphed. Let's choose several points with integer coordinates to help us determine the slope of the line. In each case, we will note the horizontal distance by a red segment and the vertical distance by a green segment.





When we compare the change in the y-value (2) to the change in the x-value (1) for each "slope triangle", we have the ratio $\frac{2}{1}$, which is the slope for the line.

Using the same line, look at a different pair of slope triangles.

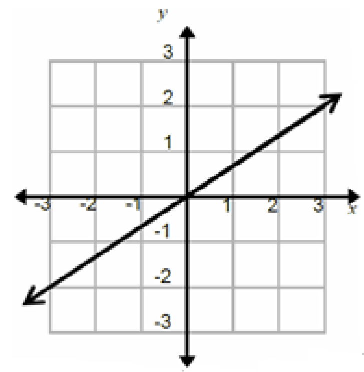
The ratio of the larger slope triangle is $\frac{4}{2}$.

The ratio of the smaller triangle is $\frac{2}{1}$.

The ratios are equivalent! $\frac{4}{2} = \frac{2}{1} = 2$

These slope triangles are similar by angle-angle (the right angle and the common angle). When we have similar triangles, we know that the ratios of the corresponding sides must be equal. That is the reason that the slope is the same for both slope triangles.

Example problems:



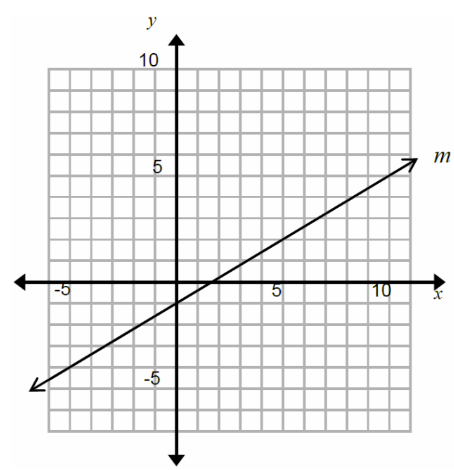
1. *Determine the slope for the line graphed to the right.*

   Change in the y-value is 2; change in the x-value is 3. Therefore, the slope is $\frac{2}{3}$.

2. *Start at (0, 0). Move right 6 units. From the location defined by completing these steps, how many units up is the line?*

   Using the slope, move 3 more units to the right and up 2 units, identifies another point on the line $(6, 4)$. Therefore, the answer is 4 units.

Check Yourself: Show another way to show that the slope of a line is constant by using similar triangles



1. On the graph to the right, plot four points with integer coordinates on the line m. Label the points P, Q, R, and S.
2. Draw the slope triangle using points P and Q. Label the right-angle vertex T.
3. Draw the slope triangle using points R and S. Label the right-angle vertex V.
4. Extend $\overleftrightarrow{PT}$ and $\overleftrightarrow{RV}$ to create horizontal (and parallel) lines across the coordinate plane.
5. When we have similar triangles, we know that the ratios of the corresponding sides must be equal. Therefore, the slope of the line is constant. What is the slope of this line ______.
6. When we have similar triangles, we know that the ratios of the corresponding sides must be equal. Therefore, the slope of the line is constant. What is the slope of this line?

Deriving $y = mx$

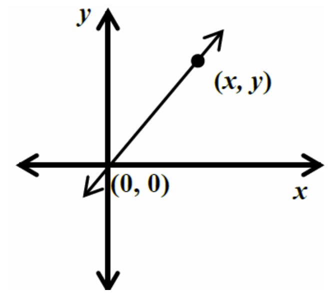
We know that the graphs for direct variation always go through the origin (0, 0). Knowing that, let's derive the equation for direct variation.

$$\frac{y_2 - y_1}{x_2 - x_1} = m \qquad \text{slope formula}$$

$$\frac{y-0}{x-0} = m \qquad (x_1, y_1) = (0,0) \text{ and } (x_2, y_2) = (x, y)$$

$$\frac{y}{x} = m \qquad \text{simplify}$$

$$y = mx \qquad \text{Multiplication Property of Equality}$$

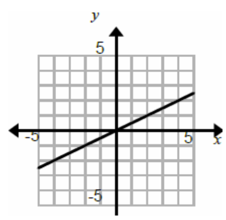


So, in a *direct variation equation*, $y = mx$, the m represents the constant of proportionality (variation), the slope and the unit rate.

Example: *Which functions show a proportional relationship? How do you know?*

x	0	1	2
y	0	3	6

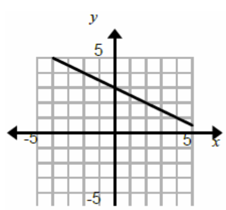
Yes, passes through (0, 0)



Yes, passes through (0, 0)

$\boxed{y = -2x}$

Yes, $y = mx$



No, does not pass through (0, 0)

$\boxed{y = 2x - 3}$

No, not $y = mx$

x	3	6	9
y	1	2	3

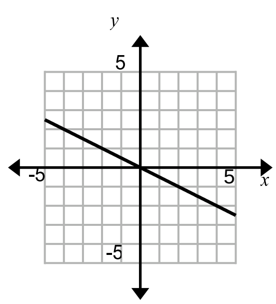
Yes, passes through (0, 0)

Check Yourself: Which functions show a proportional relationship? How do you know?

1.

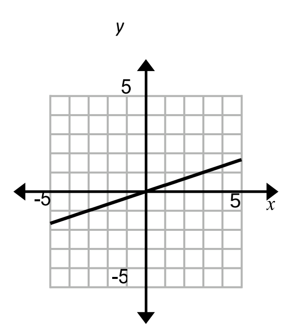
x	0	1	2
y	0	4	8

2.



3. $\boxed{y = x}$

4.



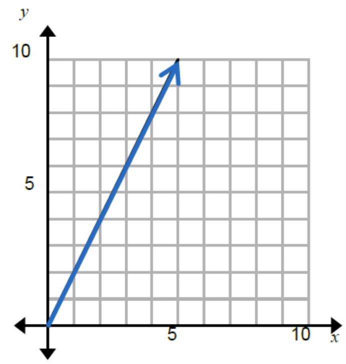
Interpreting the *y*-intercept

Create a chart and graph for the scenarios below. Identify the slope.

A. *Each week Marlow puts 2 dollars away per week to save up some money to buy a new video game.*

Slope: $\frac{2\ dollars}{1\ week}$

Week	Total Money Saved ($)
0	0
1	2
2	4
3	6



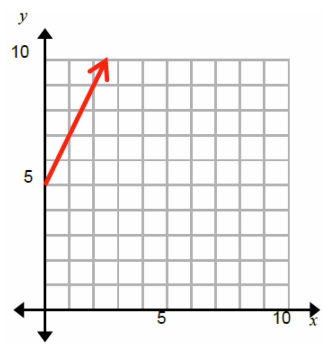
B. *Each week Marlow's friend, Jayden, puts 2 dollars away per week to save up some money to buy a new video game—the same as Marlow. However, she already had 5 dollars when they started.*

Slope: $\frac{2\ dollars}{1\ week}$

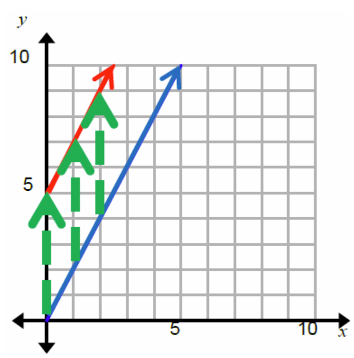
+1

Week	Total Money Saved ($)
0	5
1	5+2=7
2	5+4=9
3	5+6=11

+2



Note the slope of the lines. They are the same! Each step horizontally is 1 week. Each step vertically is 2 dollars. Putting the graphs on the same grid should make it clear that Jayden's line is a vertical translation of Marlow's line by 5 units.

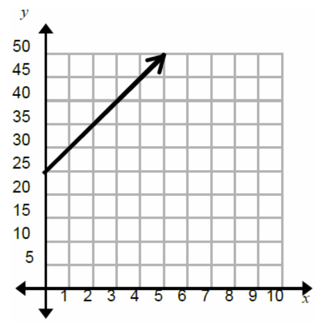


Note that the relationship represented by Marlow's savings is proportional, so we know the equation for Marlow's line is $y = 2x$. But the relationship represented by Jayden's is not proportional, so what is the equation for her line? The rate of change is the same for both graphs, so we know the slope is 2. Every point on Jaydens' line is a vertical translation of Marlow's by 5.

Therefore, $y = 2x + 5$. The slope m (value of 2 in this problem) represents the rate of change, and the initial value of 5 is labeled ***b*** and called the y-intercept**.** Non-proportional linear relationship can be written in the form $y = mx + b$ called the *slope-intercept form*; m is the slope and b is the y-intercept.

Let's look at a similar problem:

The senior class is selling t-shirts for homecoming week. It costs \$25 for the original design and then \$5 to print each shirt. Show the graph for this scenario and write an equation.



The graph intersects at 25 (initial cost), so b $= 25$. *Slope (m) is* $\frac{5}{1} = 5$.

Using the slope intercept form
$$y = mx + b$$
$$y = 5x + 25$$

Deriving $y = mx + b$

Complete the steps to derive the equation for a non-proportional linear relationship by using the slope formula.

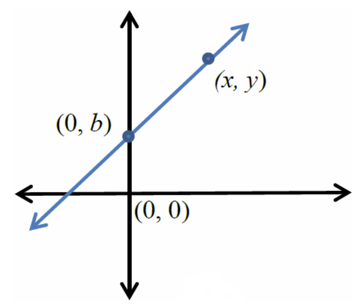
$\frac{y_2 - y_1}{x_2 - x_1} = m$ slope formula

$\frac{y - b}{x - 0} = m$ $(x_1, y_1) = (0, b)$ *and* $(x_2, y_2) = (x, y)$

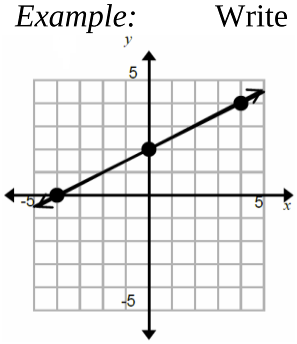
$\frac{y - b}{x} = m$ simplify

$y - b = m \cdot x$ Multiplication Property of Equality

$y = mx + b$



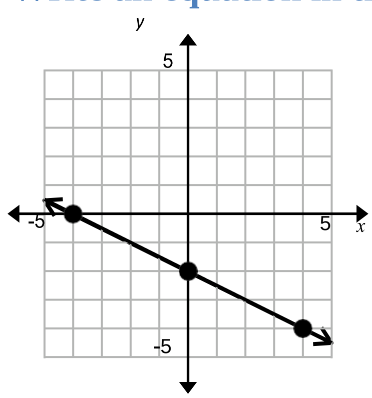
Example: Write an equation in the slope-intercept form for the graph shown.



The y-intercept is 2. From (0, 2), you can move up 2 units and to the right 4 units to reach another point on the line. That makes the slope $\frac{2}{4} = \frac{1}{2}$

$$y = mx + b$$
$$y = \frac{1}{2}x + 2$$

Check Yourself: Write an equation in the slope-intercept form for the graph shown.



DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- MAKE LINES FROM RIGHT TRIANGLES
- DERIVE "$y = mx$" USING SIMILAR TRIANGLES
- DERIVE "$y = mx + b$" USING SIMILAR TRIANGLES

TAKE THE **MAFS.8.EE.2.6** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.EE.2.6**

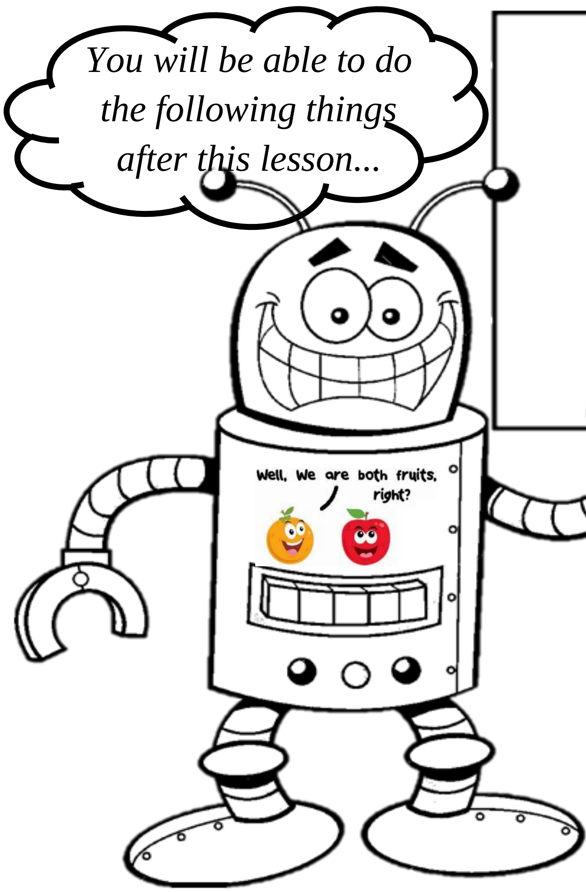
I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

- COMPLETE LESSON **MAFS.8.EE.2.6** ON EDGENUITY
- GO TO LESSON 7: COMPARE PROPERTIES OF TWO FUNCTIONS

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 7: COMPARE PROPERTIES OF TWO FUNCTIONS

Lesson 7: Compare Properties of Two Functions



- Compare and contrast two functions with different representations.
- Draw comparisons base on different representations of functions.

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

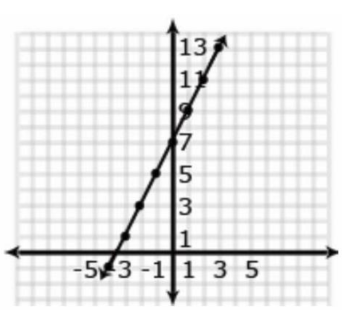
- Multiple Representations

Comparing Properties of Two Functions

The following is adapted from the Arizona Academic Content Standards, 2010.

Example: *Compare the two linear functions listed below and determine which function represents a greater rate of change.*

Function 1:



Function 2:

The function whose input x and output y are related by $y = 3x + 7$

Solution:

Function 1 has a slope (rate of change) of $\frac{2}{1}$ *or* 2. *Function 2 has a slope of* 3. *The greater the slope, the greater the rate of change, so Function 2 has the greater rate of change. (Steeper slope indicates greater rate of change.)*

Example: *Compare the two linear functions listed below and determine which has a negative slope*

<u>Function 1: Gift Card</u>

Samantha starts with \$20 on a gift card for the book store. She spends \$3.50 per week to buy a magazine. Let *y* represent the amount remaining as a function of the number of weeks, *x*.

x	y
0	20
1	16.50
2	13.00
3	9.50
4	6.00

<u>Function 2:</u>

The school bookstore rents graphing calculators for \$5 per month. It also collects a non-refundable fee of \$10.00 for the school year. Write the rule for the total cost (c) of renting a calculator as a function of the number of months (*m*).

Solution:

Function 1 is an example of a function whose graph has negative slope. Samantha starts with \$20 and spends money each week. The amount of money left on the gift card decreases each week. The graph has a negative slope of -3.5, which is the amount the gift card balance decreases with Samantha's weekly magazine purchase. Function 2 is an example of a function whose graph has positive slope. Students pay a yearly nonrefundable fee for renting the calculator and pay \$5 for each month they rent the calculator. This function has a positive slope of 5 which is the amount of the monthly rental fee. An equation for Example 2 could be $c = 5m + 10$.

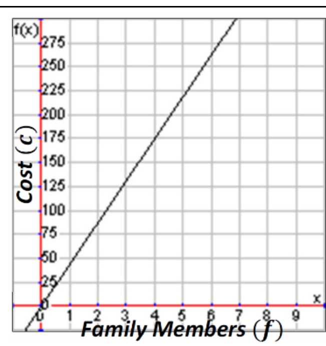
Check Yourself: Answer the following questions comparing function equations, graphs, tables and descriptions.

Your family is deciding which activity to participate in while on your vacation in San Diego. Here is the information about the cost (*c*) for admission for all your family members (*f*). Included in the cost is the parking fee for each.

City Tour
Charges \$40 per family member plus a \$20 parking fee

San Diego Zoo
Cost is modeled by the equation c = $\frac{85}{2}f + 10$

SeaWorld



Kayaking

f	2	4	6	8	10
c	75	125	175	225	275

1. Which activity is the cheapest per family member, and how do you know?

2. Which activity is the most expensive per family member, and how do you know?

3. Which activity has the cheapest parking fee, and how do you know?

4. Which activity has the most expensive parking fee, and how do you know?

5. How many of your family members could you bring to each activity if you budgeted $200?

6. Which activity allows you to bring the most people for that amount of money?

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- COMPARE TWO FUNCTIONS BY ANALYZING AN EQUATION AND A GRAPH
- COMPARE TWO FUNCTIONS BY ANALYZING AN EQUATION AND A TABLE
- COMPARE TWO FUNCTIONS BY ANALYZING AN EQUATION AND A VERBAL DESCRIPTION

TAKE THE **MAFS.8.F.1.2** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.F.1.2**

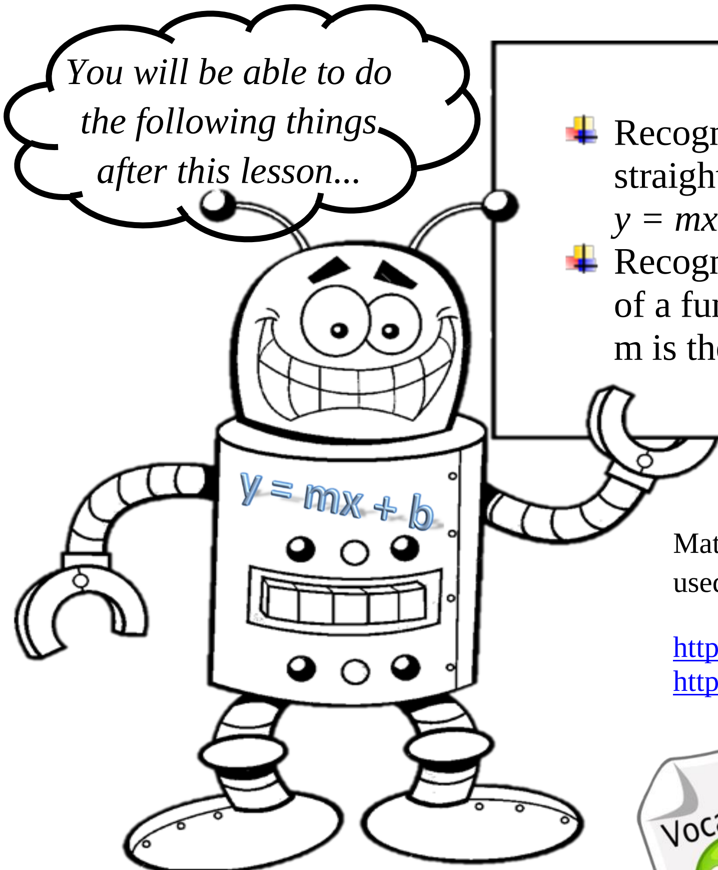
I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

- COMPLETE LESSON **MAFS.8.F.1.2** ON EDGENUITY
- GO TO LESSON 8: SLOPE-INTERCEPT FORM OF LINEAR EQUATION

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 8: SLOPE-INTERCEPT FORM OF LINEAR EQUATION

Lesson 8: Slope-Intercept Equation of a Line



- Recognize that a linear function is graphed as a straight line represented as an equation in the form $y = mx + b$.
- Recognize the equation $y = mx + b$ is the equation of a function whose graph is a straight line where m is the slope and b is the y-intercept

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- Slope-intercept form
- y-intercept

Slope-Intercept Form of a Linear Equation

Now for the shortcut….

We have $3x - 4y = 12$. Solving for y, we have $y = \frac{3}{4}x - 3$. We graphed that and found the graph to cross the y-axis at (0, -3). We can determine the slope by using two of the points:

$$m = \frac{-3 - 0}{0 - 4} = \frac{-3}{-4} = \frac{3}{4}$$

I know the graph will cross the y-axis at (0, -3) and has a slope of $\frac{3}{4}$. That means I can locate the point the graph crosses the y-axis, called the *y-intercept*; from there, go up 3 units and over 4 units to locate another point on the line. We can now draw the line.

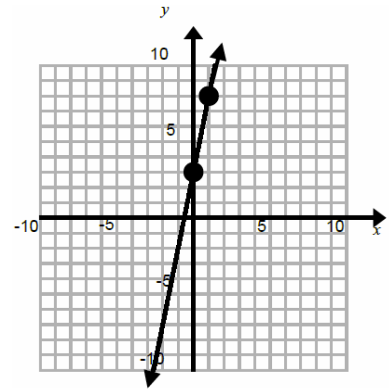
If I did enough problems like that, I would begin to notice the coefficient of the *x* is the *slope* and the constant is the *y-intercept* (where the graph crosses the y-axis). That leads us to the

Slope-Intercept Form of an Equation of a Line:

$$\boldsymbol{y = mx + b}$$

where $\boldsymbol{m}$ *is the slope and* $\boldsymbol{b}$ *is the y-intercept.*

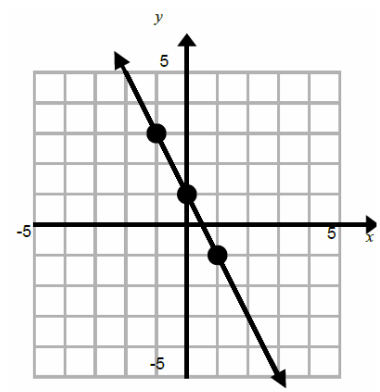
Let's look at another problem, $y = 5x + 3$.



To graph that, I would find the graph crosses the *y*-axis at (0, 3) and has slope 5. That means, I would locate the point the graph crosses the *y*-axis, called *the y-intercept (0, 3)*, and from there go up 5 spaces and over one $\left(recognizing\ slope\ as\ \frac{rise}{run} = \frac{5}{1}\right)$

The graph always crosses the *y*-axis when *x* = 0.

Example: *Find the y-intercept and slope of* $y = -2x + 1$. *Graph.*

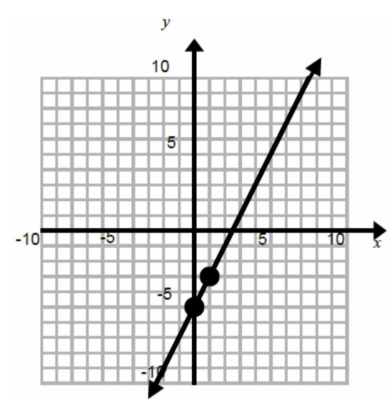


Without plugging in *x*'s and finding *y*'s, I can graph this by inspection using the slope-intercept form of a line.

The equation $y = -2x + 1$ is in the $y = mx + b$ form. In our case, the *y*-intercept is 1, written (0, 1), and the slope is -2. This slope can be written as $\frac{-2}{1}$ or $\frac{2}{-1}$

To graph, I begin by plotting a point at the *y*-intercept (0, 1). Next, the slope is -2. So, from the *y*-intercept, I go down two and to the right one and place a point. (Or I could have gone up two and to the left one.) Connect those points, and I have my line.

Example: *Find the y-intercept and the slope of* $y = 2x - 5$ *Graph.*

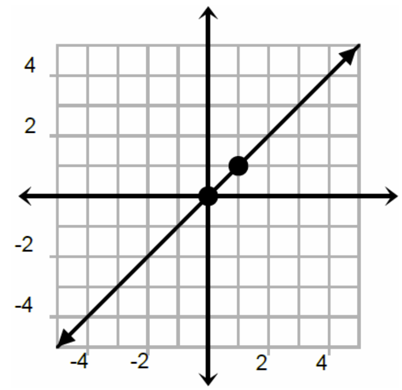


The graph of that line would cross the *y*-axis at the *y- intercept* (0, –5) and has *slope* 2.

To graph that, I would go to the *y*-intercept (0, –5) and from there, go up two spaces and to the right one space.

The question comes down to: would you rather learn a shortcut for graphing or do it the long way by solving for *y* and plugging in values for *x*? Let's try one more.

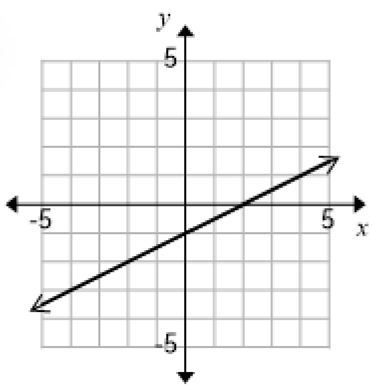
Example: *Graph* $y = x$.



The graph of that line would cross the *y*-axis at (0, 0) and has slope 1.

To graph that, I would go to the *y*-intercept (0, 0) and from there, go up one space and over one.

Check Yourself: Select the equation represented by the graph.



A. $y = \frac{1}{2}x - 1$

B. $y = -\frac{1}{2}x - 1$

C. $y = 2x - 1$

D. $y = -2x - 1$

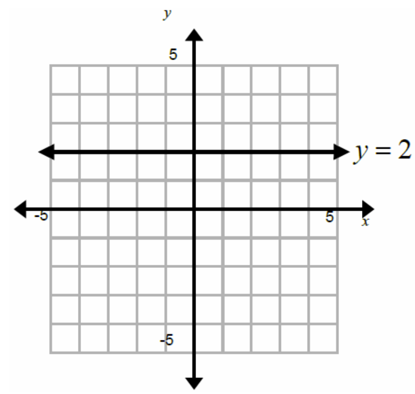
Horizontal and Vertical Lines

Don't forget to address graphing *horizontal* and *vertical* lines. The graph of the equation $y = b$ is the horizontal line through $(0, b)$. The graph of the equation $x = a$ is the vertical line through $(a, 0)$. Of course, remember to simply set up a table of values if you are confused.

- *If the line is* ***horizontal****, the slope is* ***zero****.*
- *If the line is* ***vertical****, the slope is* ***undefined****.*

Example: *Graph* $y = 2$

The graph of the equation $y = 2$ is the horizontal line through $(0, 2)$.



Or a quick table would give us points to plot, and then we could draw the line.

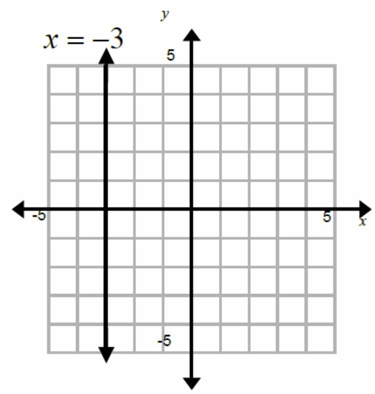
x	y
-	2
0	2
2	2

$$m = \frac{2-2}{2-0}$$

$$m = \frac{0}{2}$$

Example: *Graph* $x = -3$.

The graph of the equation $x = -3$ is the horizontal line through $(0, -3)$.



Or a quick table would give to plot and then draw the line.

x	y
–	–
–	0
–	2

$$m = \frac{0-(-2)}{-3-(-3)}$$

$$m = \frac{2}{0}$$

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- ANALYZE GRAPHS OF Y = X AND Y = MX
- ANALYZE TABLES OF Y = X AND Y = MX
- ANALYZE GRAPHS AND TABLES OF Y = MX + B

TAKE THE **MAFS.8.F.1.3** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.F.1.3**

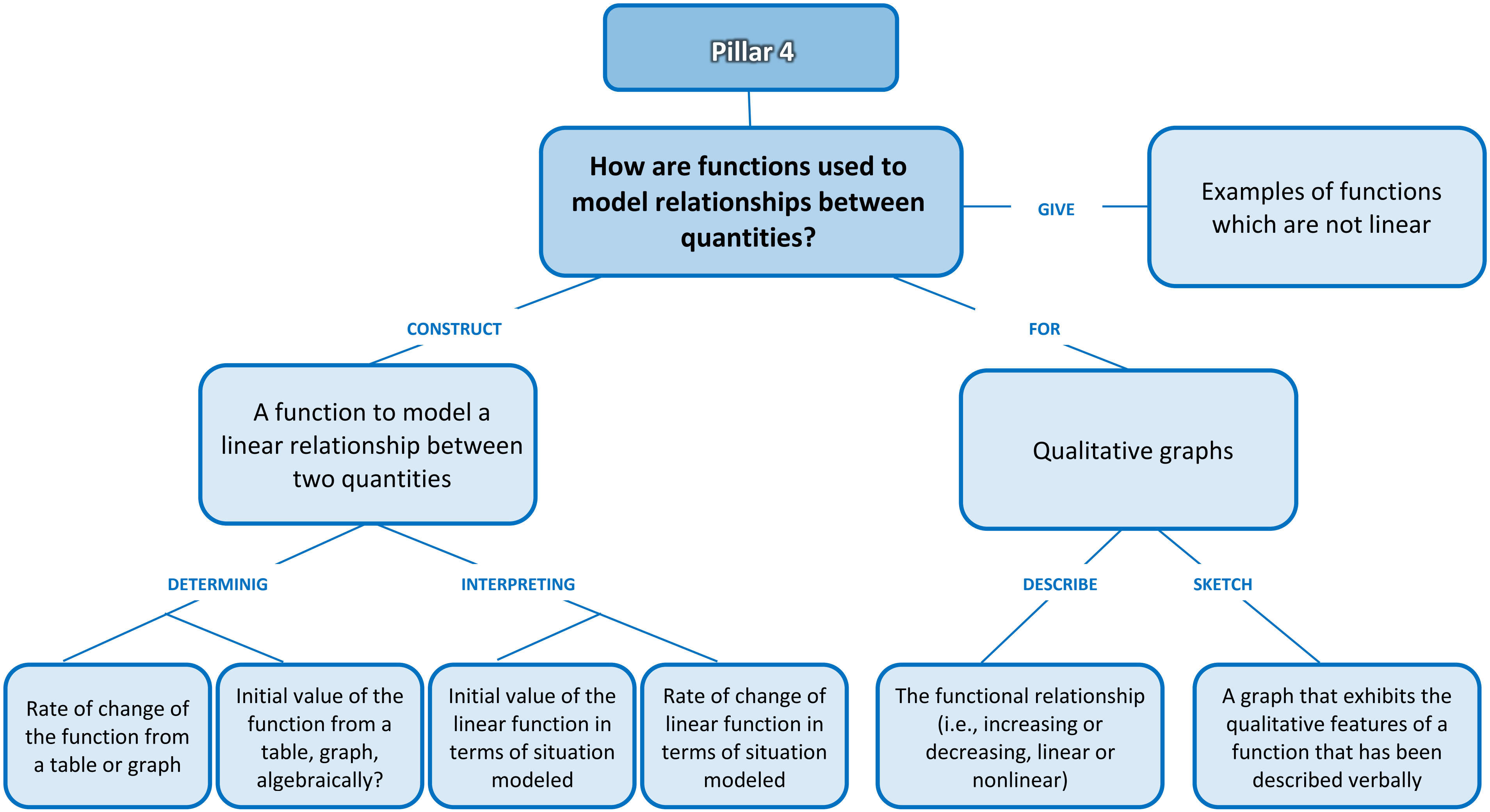
I SCORED BELOW 70% ON THE EDGENUITY

- COMPLETE LESSON **MAFS.8.F.1.3** ON EDGENUITY
- GO TO LESSON 9: INVESTIGATING LINEAR FUNCTIONS

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

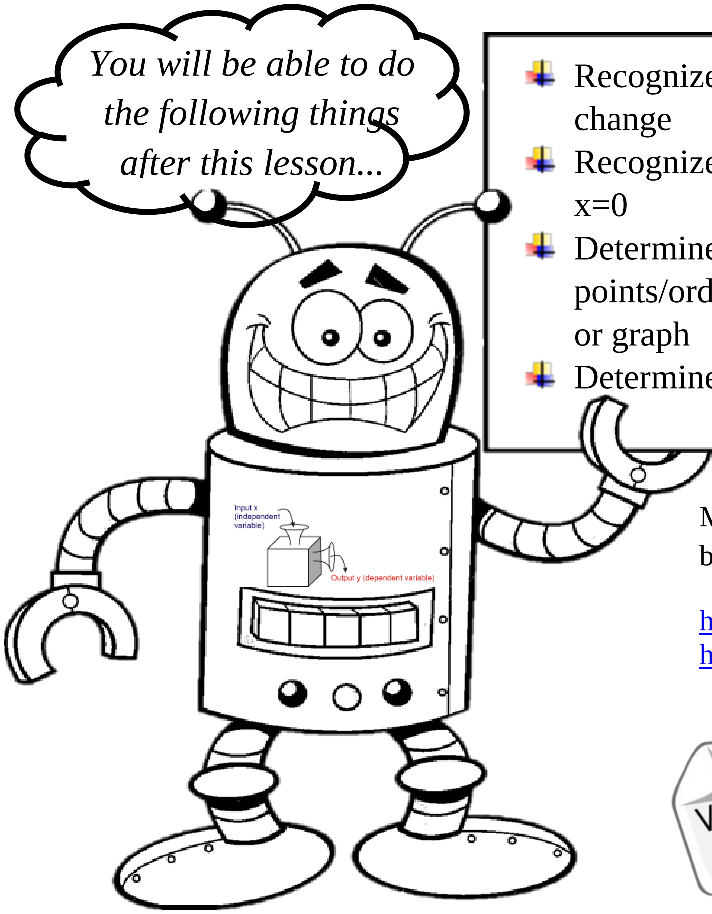
- GO TO LESSON 9: INVESTIGATING LINEAR UNCTIONS

PILLAR #4



Algebra 1: Functions »Linear and Exponential Functions

Lesson 9: Investigating Linear Functions



- Recognize that slope is determine by the constant rate of change
- Recognize that the y-intercept is the initial value where x=0
- Determine the rate of change (slope) from two coordinate points/ordered pair, a verbal description, values in a table, or graph
- Determine the initial value (y-intercept) from two

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

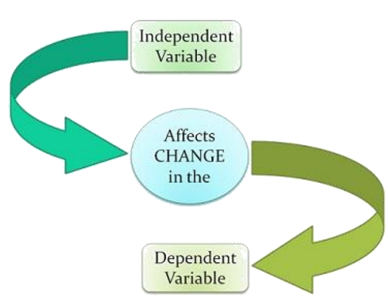
https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- Slope
- y-intercept
- Rate of Change
- Initial value

Exploring Linear Functions

Input (Independent Variable) and Output (Dependent Variable)

Since a function is a rule that assigns each input exactly one output, it is crucial to identify and understand what the input and output are in the multiple forms of a function. Let's look at the following linear functions written in different forms.



Example 1

The number of grapes (g) depends on the number of branches (b) off the main vine and is represented by this equation:

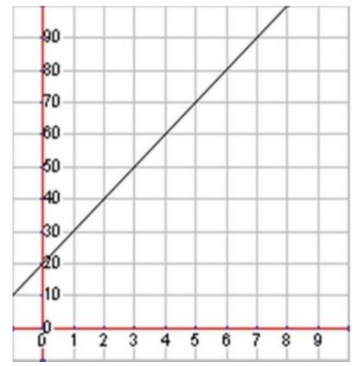
$$g = 100b + 4$$

Example 2

A dairy farmer can produce 25 gallons of milk (g) from 3 cows (c).

Example 3

The cost (c) for joining a gym includes a start-up fee plus dues every month (m).



In Example l, what is the input? What do we start with? What comes first? The branches of the vine. Once we have branches, those branches then produce grapes. So, the input, the thing we start with, is the independent variable b.

The independent variable is the variable that could be anything (at least anything within the domain). We could have any number of branches we want and the branches produce, or output, the grapes. That means that g is the dependent variable. The number of grapes depends on the number of branches.

In Example 2, what is the input and what is the output? A farmer puts cows in his barn and gets out milk. The cows are the input meaning that c is the independent variable. The milk is the output meaning that g is the dependent variable.

In Example 3, what is the input and what is the output? What do we really want to know? The final cost. However, to find the cost we first have to know how many months you are going to be a member. That means that the number of months is the input, or independent variable. Once we input the number months into the rule (which happens to be times l5 and then plus 20), we output the cost, which is the dependent variable.

In the standard equation form of a linear function, $y = mx + b$, what is the input and output? Since we're talking about x and y on the coordinate plane, those are my input and output, but which is which? But not always, the output is the variable by itself in any equation. In our generic form linear function, the variable y is the output. That makes x the input. If we plug in (input) an x-value then we get out (output) a y-value.

Slope or Rate of Change

Let's look at our examples again

Example 1

The number of grapes (g) depends on the number of branches (b) off the main vine and is represented by this equation:

$$g = 100b + 4$$

Example 2

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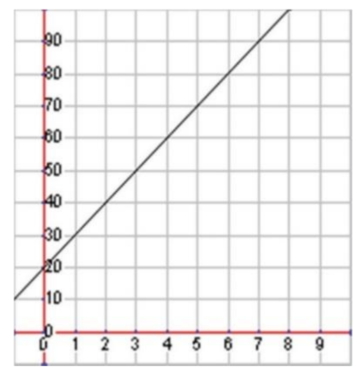
The cost (c) for joining a gym includes a start-up fee plus dues every month (m).



In Example 1, the rate of change is given in the equation as the slope or m in the equation $y = mx + b$. Note that the slope is 100 which means that the rate of change is also 100, or $\frac{100}{1}$ in fraction form. This means that 100 grapes grow for every one branch.

Perhaps it is easiest to see the rate of change (or slope) in the second example. How does that amount of milk change for the farmer? He gets 25 more gallons of milk for every 3 more cows he has, so we would write that rate of change as $\frac{25}{3}$.

In the third example, we'll need to find the slope by counting the rise and run. This is easiest to do from the y-intercept, the point where the line crosses the y-axis. Notice that the line crosses the y-axis at 20. The next nice point is at (l, 30). To get to that point from the y-intercept, you have to go up ten and right one. That means the slope, or rate of change, is 10. This means that the gym charges 10 dollars for every one month of membership.

"I wonder if our rate is proportional to the slope of the hill."

y- Intercept or Initial Value

Let's look at our examples again.

Example 1

The number of grapes (g) depends on the number of branches (b) off the main vine and is represented by this equation:

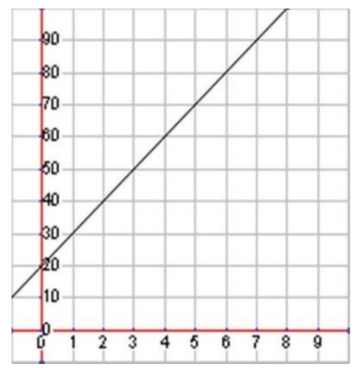
$$g = 100b + 4$$

Example 2

A dairy farmer can produce 25 gallons of milk (g) from 3 cows (c).

Example 3

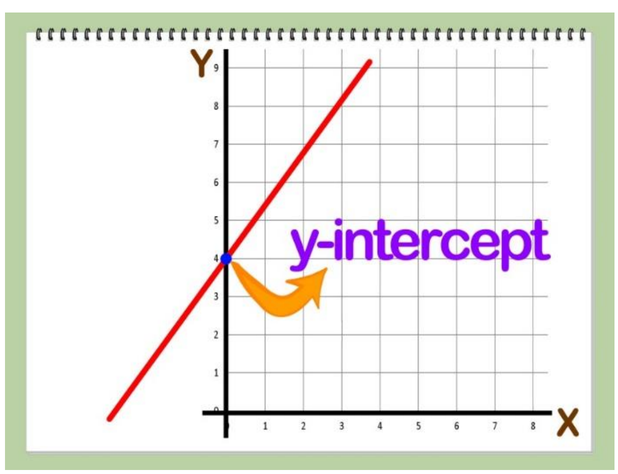
The cost (c) for joining a gym includes a start-up fee plus dues every month (m).



In Example 1, the initial is given in the equation as the y- intercept, or b value in y = mx + b. Note that the initial value is 4. This means that only 4 grapes grow off the main vine no matter how many branches come off the main vine.

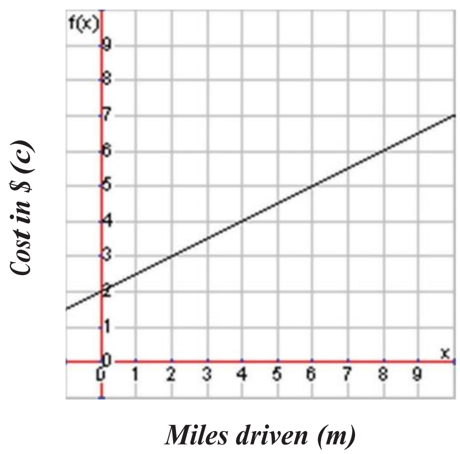
The second example may be confusing because it's hard to see an initial value. An initial value would mean the amount of milk that the farmer starts with. Well, without any cows he wouldn't have any milk, so the initial value is 0. That's why there is no other number mentioned.

We previously found the y-intercept for the third example to be 20. That means that no matter how many months you pay for membership to the gym, there will always be an additional $20 fee to pay. The problem context describes it as a start-up fee. So, if you pay for 3 months' membership, you'll pay the $20 fee on top of the price per month. If you pay for 85 months of membership, you'll still pay the same $20 fee on top of the price per month.



Practice: Identify the rate of change, initial value, independent variable, and dependent variable. Then describe what the rate of change and initial value mean in the context of each situation. Finally, write the equation of the linear function.

The cost for 6 people to travel in a taxi in New York *(c)* based on the number of miles driven *(m)* is shown by the following graph:



Contextual Description of Rate of Change	Contextual Description of Initial Value

1. What is the rate of change?

2. What is the initial value?

3. What is the independent variable?

4. What is the dependent variable?

5. What is the equation of the line?

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- DETERMINING THE CONSTANT RATE OF CHANGE
- DETERMINING THE Y-INTERCEPT
- CREATE EQUATION, TABLE, AND GRAPH FROM A SITUATION
- INTERPRETING SLOPE AND Y-INTERCEPT IN CONTEXT
- CONSTRUCT A LINEAR FUNCTION

TAKE THE **MAFS.8.F.2.4** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.F.2.4**

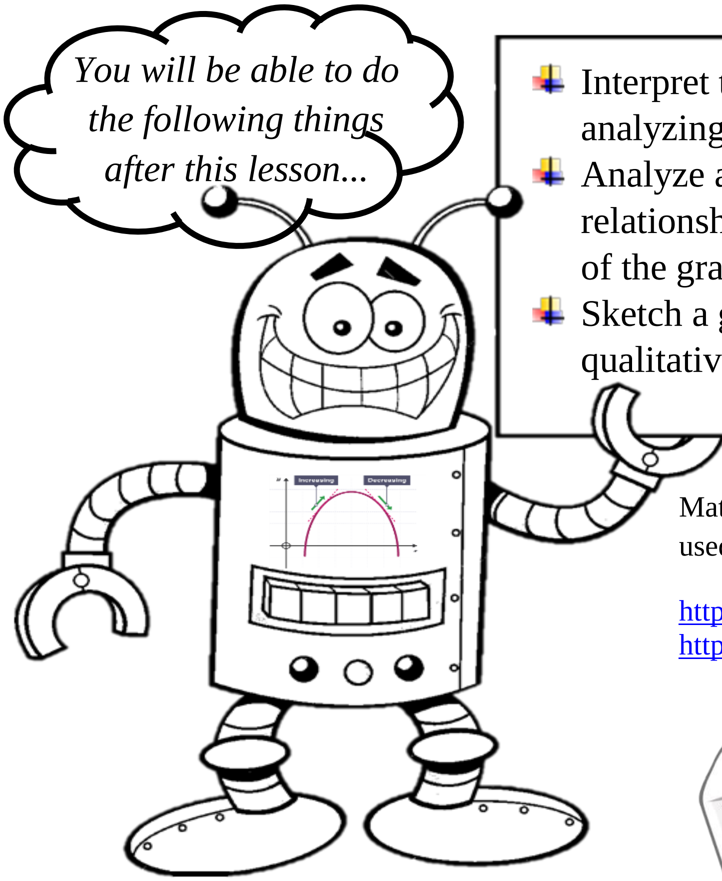
I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

- COMPLETE LESSON **MAFS.8.F.2.4** ON EDGENUITY
- GO TO LESSON 10: QUALITY FUNCTIONAL RELATIONSHIP.

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 10: QUALITY FUNCTIONAL RELATIONSHIP.

Lesson 10: Qualitative Functional Relationship



- Interpret the relationship between *x* and *y* values by analyzing a graph
- Analyze a graph and describe the functional relationship between two quantities using the qualities of the graph
- Sketch a graph given a verbal description of its qualitative features.

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- Increasing
- Decreasing
- Linear
- Non-linear
- Constant
- Interval
- Relation

Increasing and Decreasing

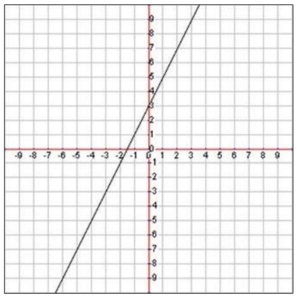
So far, we have been describing graphs using quantitative information. That's just a fancy way to say that we've been using numbers. Specifically, we have described linear function graphs using the rate of change and initial value. Both are numerical data, however, at times it is more beneficial to describe functions in a qualitative manner. This means describing the qualities, non-numerical characteristics, of the function.

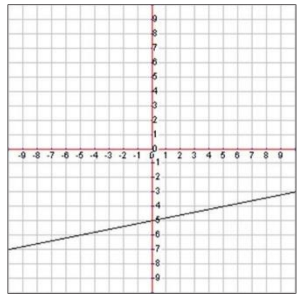
Qualitative Data: Increasing or Decreasing

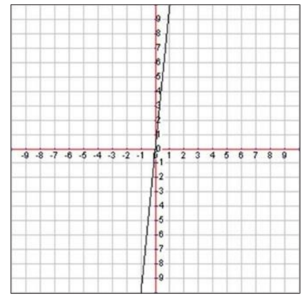
Let's start with the idea of an increasing or decreasing function. An increasing function roughly speaking is one that is going up when you look at it from left to right. This means that a decreasing function is one that is going down when you look at it from left to right. Let's start by looking at linear functions.

Linear functions

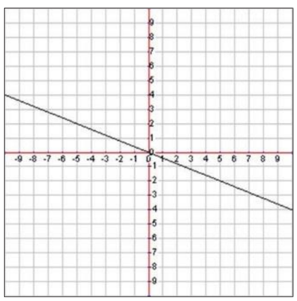
Increasing Linear Functions

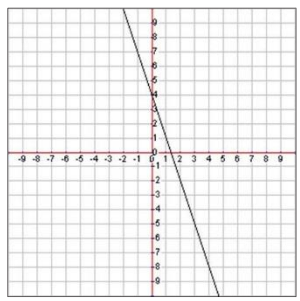


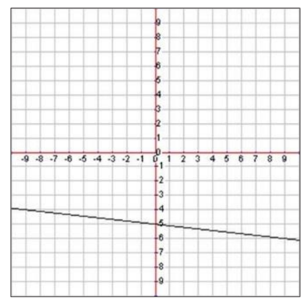




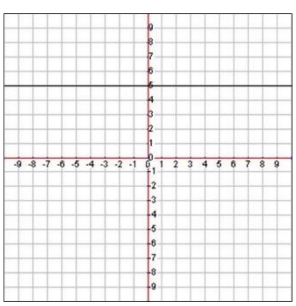
Decreasing Linear Functions







It is worth noting that in linear functions, whether it is increasing or decreasing is dependent on the slope of the line. Notice that those with positive slopes are increasing and those with negative slopes are decreasing.

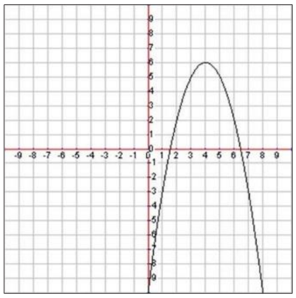


However, how would we classify the graph to the left? It is possible to have a third option rather than just increasing or decreasing. It could be neither. The graph to the left is called constant because it stays the same from left to right.

It would be natural to then ask about a vertical line, but remember that a vertical line would not pass the vertical line test and therefore would not be a true function.

Non-linear functions

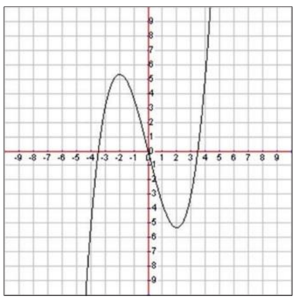
Linear functions are easy to identify as increasing, decreasing, or constant because they are a straight line. Non-linear functions might be both increasing and decreasing at different points on the graph. Consider the following graph of the function $y = -x^2 + 8x - 10$.



Looking at this graph from left to right, we see that the graph starts out increasing, but then it reaches a high point and start decreasing. So how do we classify this function?

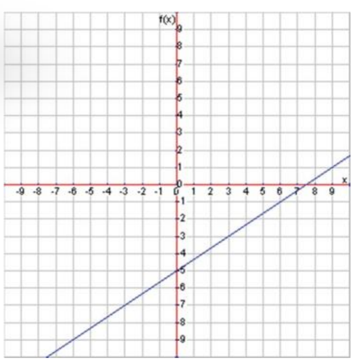
Let's see if we can isolate where the function is increasing and just look at that piece of the graph. Notice that the graph is increasing until it reaches the point at $x = 4$. That means we can say that the function is increasing when $x < 4$. In other words, whenever x is less than 4, the graph is increasing.

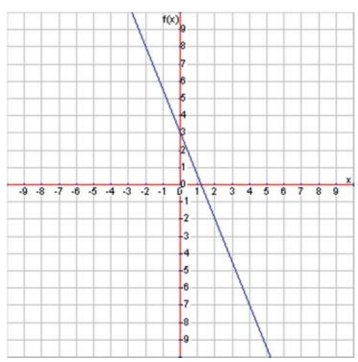
We can similarly see where the graph is decreasing which when $x > 4$. Notice that we don't use greater than or equal to, just greater than because at the exact point $x = 4$ the graph is at the high point and neither increasing or decreasing. We'll get to what that point is called in just a moment.

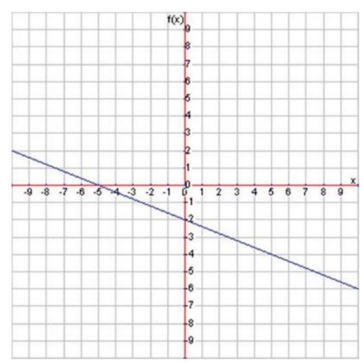


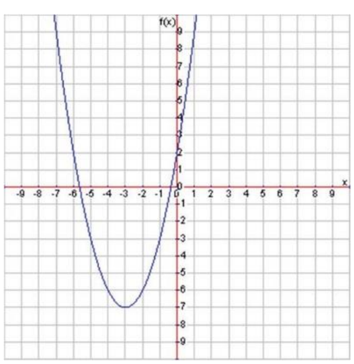
To the left is the graph of the function $y = \frac{1}{3}x^3 - 4x$. Notice that it's increasing, then decreasing, then increasing again. So, we would say it is increasing when $x < -2$ and $x > 2$. It is decreasing in the interval $-2 < x < 2$.

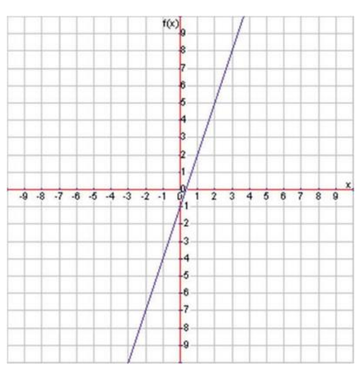
Practice: Which graph represents a linear function increasing at a constant rate? Select all that apply by filling in the appropriate box.

☐ 

☐ 

☐ 

☐ 

☐ 

Sketching a Piecewise Function

Now that we understand qualitative descriptions of graphs, we can use that information to sketch graphs of a function or give a verbal description of an already sketched graph. For these graphs, we won't have any numeric reference points to go by. Instead we'll just use Quadrant I of the coordinate plane and give approximate graphs that represent the described situation.

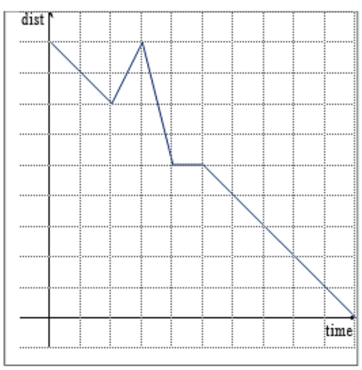
The term piecewise means that the function may have different qualities at different intervals. For example, the graph may start off constant, then increase and finally decrease. It could start increasing linearly and then increase in a non-linear fashion. So we generally sketch the graph a piece at a time.

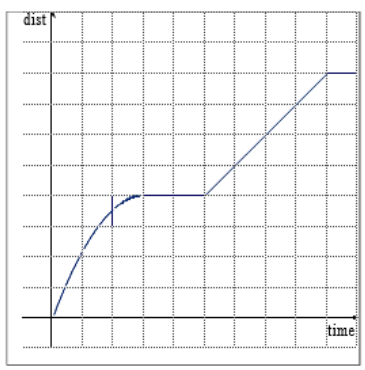
Matching Description and Graph

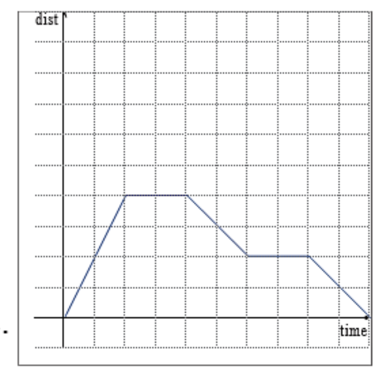
Distance versus time

We'll begin by reading a description of a situation and then decide which graph best fits the data. Here is our situation:

> George started at his friend's house and began walking home. After a few blocks, he realized he forget his cell phone and hurried back to his friend's house to pick it up. After grabbing his phone, he immediately began running back home because he was afraid he was going to be late. Unfortunately, he got stuck for a little while trying to cross the busy street. After crossing the busy street, he decided to walk the rest of the way home instead of running. Which graph shows George's distance from home in terms of time?

A. 

B. 

C. 

In this case, the choice may be obvious because only one of the three graphs start at non-zero distance. That means that graph A must be the correct graph for George. Notice how the line segments are steeper when he goes back for his cell phone and heads back home again. That's because he was running during that time so more distance was being covered in less time. Also, note the little flat line segment that shows us when George was waiting at the busy street. There he traveled no distance because he was waiting.

Using the same above graphs, match this situation with its graph:

> Joanne started running a marathon as fast as she could. During the first few minutes she gradually slowed down until she stopped at about the half-way point of the marathon. She had to take a long break sitting on a bench because she had run too fast. She then ran at a slower constant speed until the end of the marathon where she promptly collapsed. Which graph shows Joanne's distance run in terms of time?

For this situation, we're looking for a graph starting off very steep (to show the fast speed) but slowing getting less steep (to show the slowing down). Then there should be a segment where the distance does not change over time (representing a speed of zero). Afterwards we should see the distance getting bigger because Joanne completes the marathon. This must be graph B.

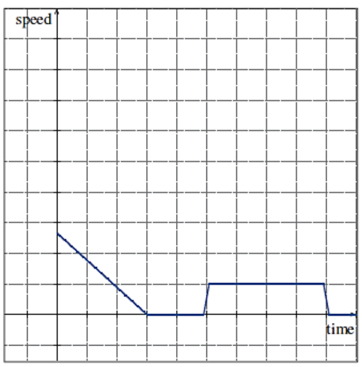
What situation might graph C represent?

Speed versus Time

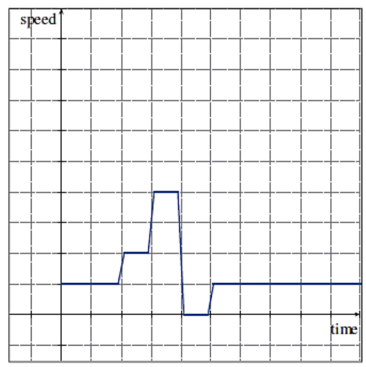
Examining these same situations in terms of speed would offer us different graphs. Let's look at George's situation first:

George started at his friend's house and began walking home. After a few blocks, he realized he forget his cell phone and hurried back to his friend's house to pick it up. After grabbing his phone, he immediately began running back home because he was afraid he was going to be late. Unfortunately, he got stuck for a little while trying to cross the busy street. After crossing the busy street, he decided to walk the rest of the way home instead of running. Which graph shows George's speed in terms of time?

D.



E.



The function showing George's speed in terms of time is represented by graph E. Notice that the speed starts off greater than zero, increases, and then increases again before going down to zero (where he stopped at the busy street) and finishing at a constant speed. More importantly notice the differences between this graph and George's graph showing distance as a function of time.

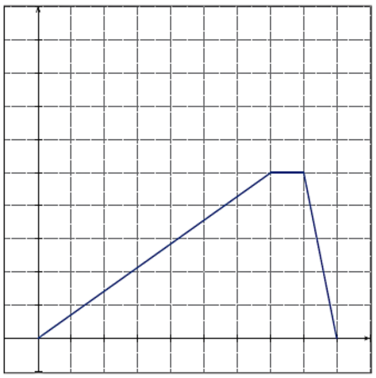
Now look at graph D. This graph represents Joanne's situation except it shows her speed instead of distance. Why is this true?

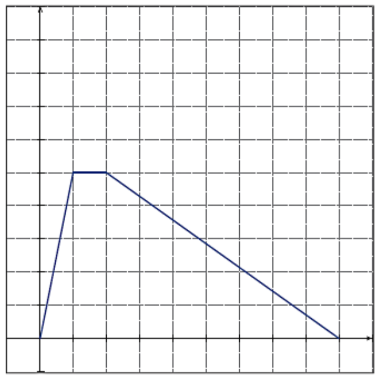
Speed or distance?

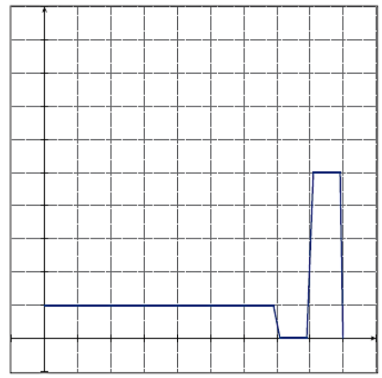
To get one last look at how different variables (speed and distance in this case) can drastically change the graph of a function, consider the following situation:

> A child climbed slowly up a slide, sat at the top for a little while, and then quickly slid down.

Which of the following graphs shows height off the ground (which is a distance) versus time and which shows speed versus time?

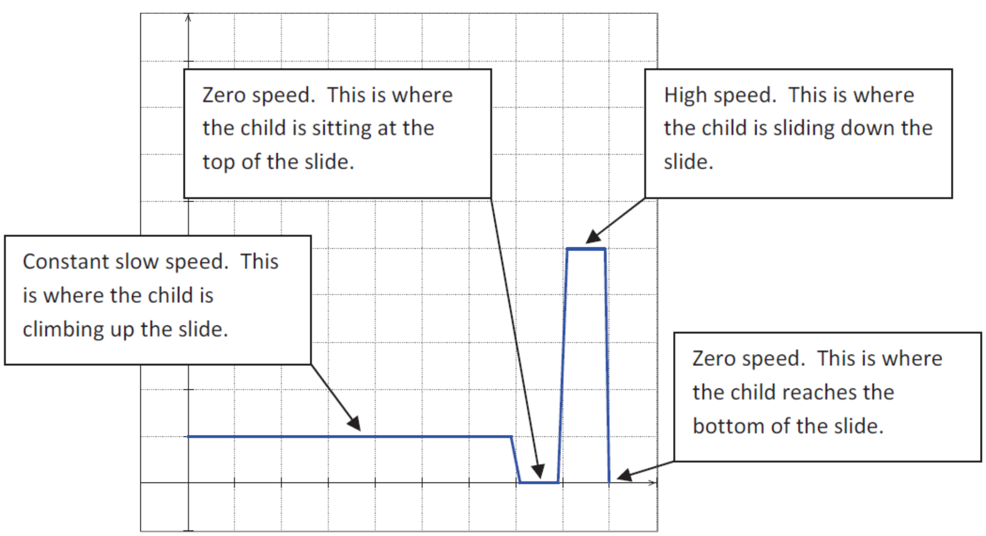
F. 

G. 

H. 

While graph G is tempting to choose as showing the height versus time because it looks like a slide, that is incorrect. Graph F shows the height as a function of time. Notice how the child takes a lot of time (horizontal distance) to get to the top of the slide and then takes far less time to have a height of zero.

Graph H is a graph of speed versus time. Let's look at why this is true.

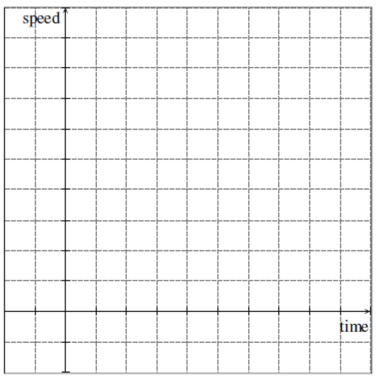


Sketching the Graph

Now let's try sketching a graph given a verbal description of a situation.

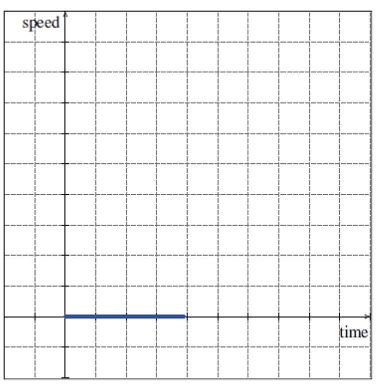
> A cat is sitting on a pillow across the room watching the ants climb up the sliding glass door. The cat sits perfectly still for several moments before quickly charging towards the sliding glass door where she slams into it coming to halt. After pausing a moment to realize the ants were scared away, the cat slowly slinks to the middle of the room to wait on the next ant to show up. Sketch a graph modeling the function of the cat's speed in terms of time.

The first thing to do is identify our two variables we are comparing and determine which is the dependent (going on the *y*-axis) and which is the independent (going on the *x*-axis). In this problem, we are looking at speed in terms of time. That means the speed is dependent on the time, and therefore speed is our dependent variable which will go on the *y*-axis. So, we might begin our graph by labeling our axes like this:

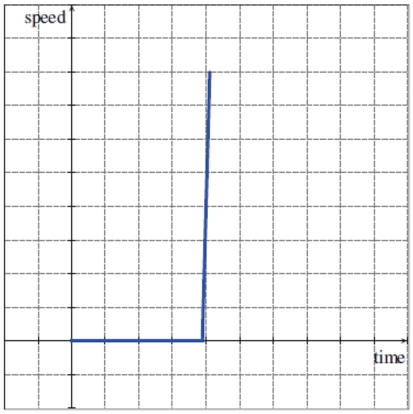


Notice we are only using Quadrant I because it doesn't make sense in this context to talk about negative time or speed.

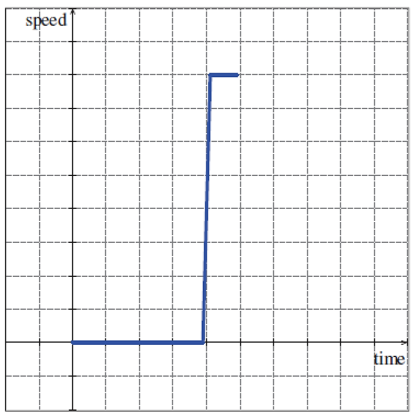
Next, we start at the beginning of the problem. What was the cat's initial speed? It was sitting perfectly still for several moments. That means for a good chunk of the time on our time, the speed will be zero. So, we'll use a flat line at height zero starting at the origin to represent this like so:



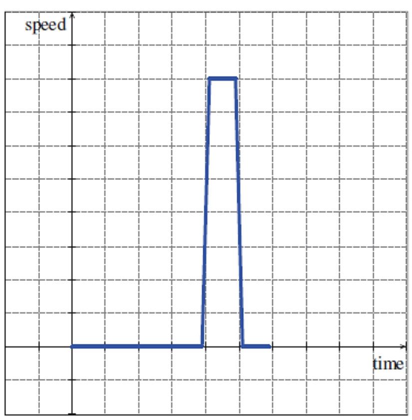
Now that we have the part of the graph representing the cat sitting still, we move to the next part of the problem. The next thing the cat does is quickly charges at the sliding glass door. That means the speed is going to be high, so we'll need a line going up to a relatively high speed.



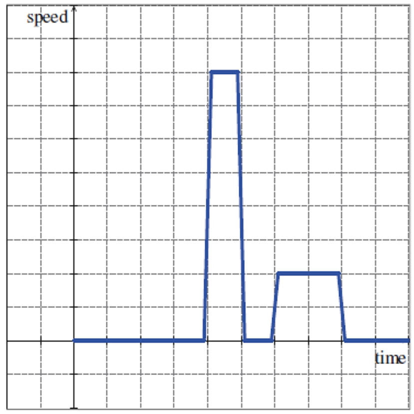
However, we also need to consider how long the cat stayed at this speed. Since it was only across the room, the cat probably did not spend a lot of time at a high speed. We'll represent this by only having the speed stay constantly high for a short amount in the *x* direction (horizontally).



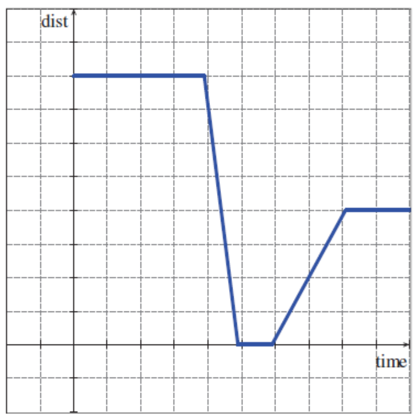
Next the cat slams into the glass door bringing its speed to zero. It also paused a moment, meaning its speed was zero just for a little bit.



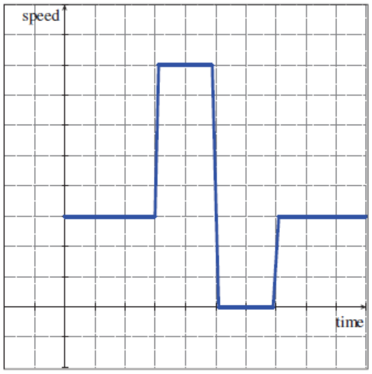
Finally, the cat slowly walks back to its pillow (low speed) before sitting back down (zero speed) in the middle of the room. So, our final graph may look like this:

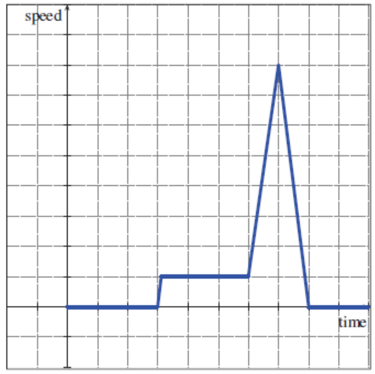


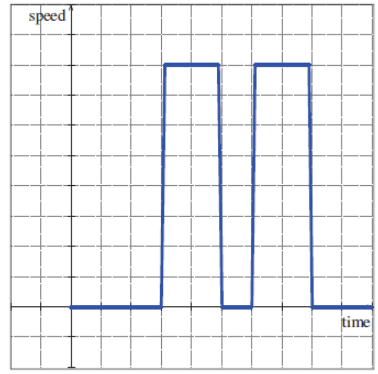
Now let's think about making a graph to represent a function of the cat's distance from the sliding glass door in terms of time. Why might it look something like this?



Practice: Match each description with its function graph showing speed in terms of time.

A.


B.


C.


1. A squirrel chews on an acorn for a little while before hearing a car coming down the street. It then runs quickly to the base of a nearby tree where it sits for a second listening again for the car. Still hearing the car, the squirrel climbs up the tree quickly and sits very still on a high branch.

2. A possum is slowly walking through a backyard when a noise scares it causing it to hurry to a hiding place. It waits at the hiding place for a little while to make sure it's safe and then continues its slow walk through the backyard.

3. A frog is waiting quietly in a pond for a fly. Noticing a dragonfly landing on the water nearby, the frog slowly creeps its way to within striking distance. Once the frog is in range, it explodes into action quickly jumping towards the dragonfly and latching onto with its tongue. The frog then settles down to enjoy its meal.

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- DESCRIBE FUNCTION RELATIONSHIPS USING GRAPHS
- EXPLAIN INTERCEPTS USING GRAPHS
- DESCRIBE RATE OF CHANGE USING GRAPHS
- SKETCH GRAPHS BY INTERPRETING SITUATIONS

TAKE THE **MAFS.8.F.2.5** EDGENUITY PRE-TEST

YES

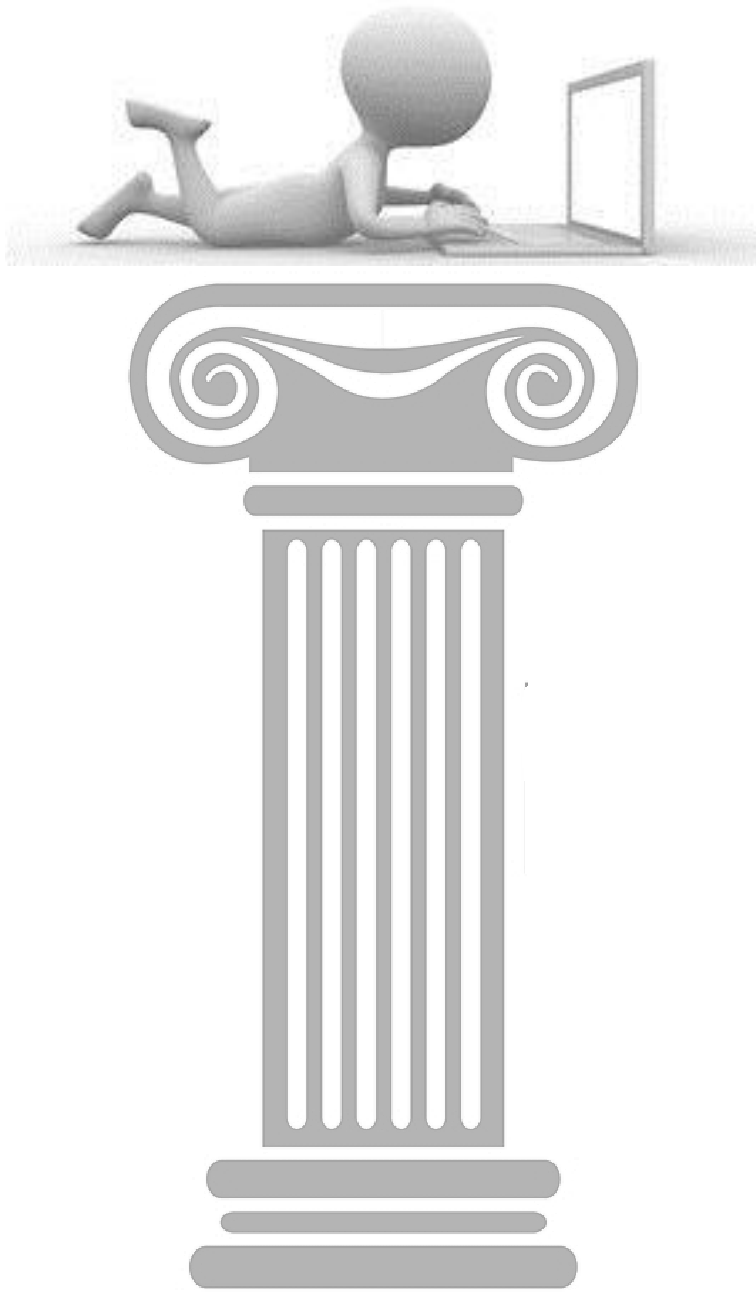
GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.F.2.5**

I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

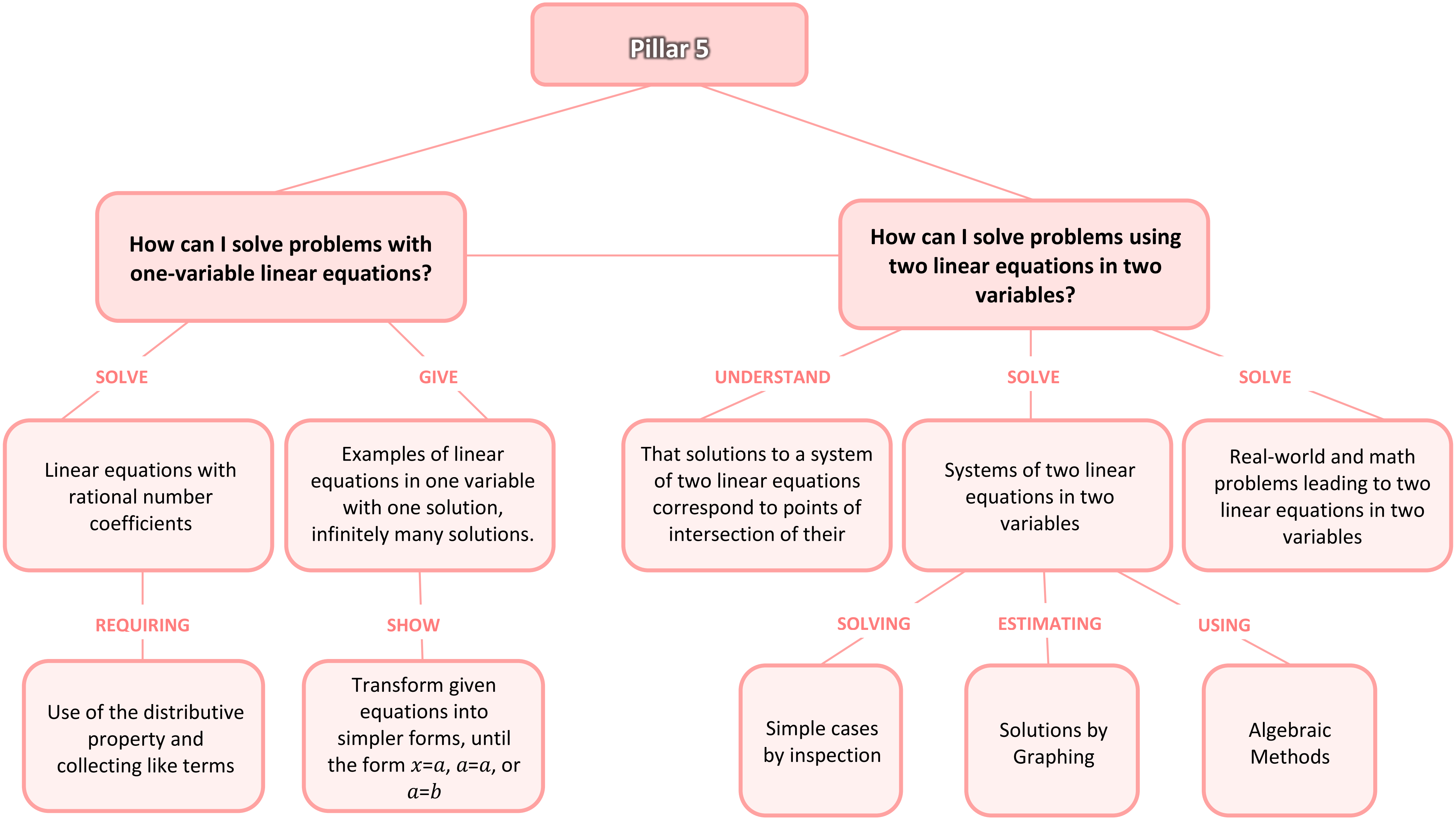
- COMPLETE LESSON **MAFS.8.F.2.5** ON EDGENUITY
- GO TO LESSON 11: LINEAR EQUATION IN ONE VARIABLE

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 11: LINEAR EQUATIONS IN ONE VARIABLE



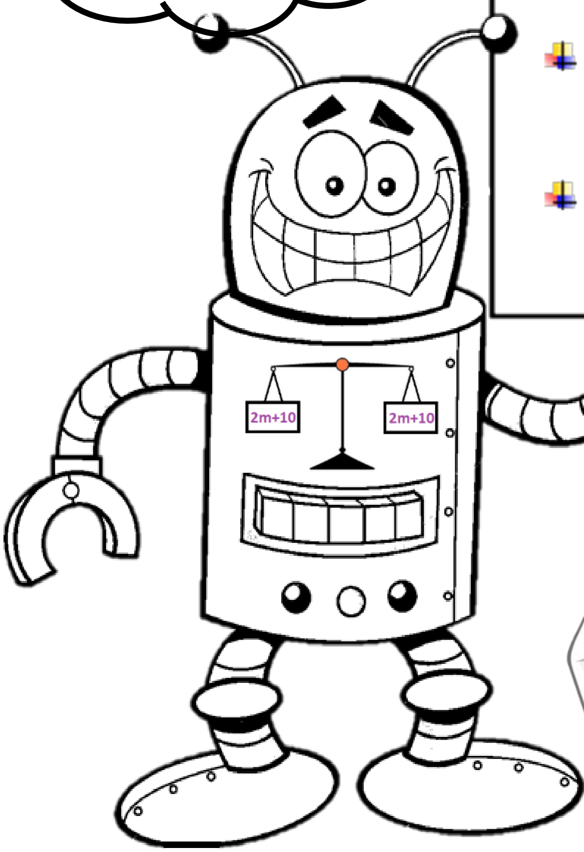
PILLAR #5



Algebra 1: Algebra » Reasoning with Equations and Inequalities

Lesson 11: Linear Equation in One Variable

You will be able to do the following things after this lesson...



- Simplify linear expressions utilizing the distributive property and collecting like terms.
- Create a multi-step linear equation to represent a real-life situation.
- Solve equations with linear expressions on either or both sides including equations with one solution, infinitely many solutions, and no solutions.
- Give examples of and identify equations as having one solution, infinitely many solutions, or no solutions.

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- Coefficient
- Equation
- Identity
- Like Terms
- Linear Expression
- Multiplicative Inverse
- Null Set
- Variable
- Properties
- Reciprocal
- Simplify
- Solution
- Term
- Two-Step Equation

Review - Properties of Real Numbers

Commutative sounds like *commute* which means to go back and forth. Addition and multiplication are both commutative; they work the same either forward or backward.

You *associate* with friends and family in groups. The Associative Properties are ways you can group addends and factors.

A teacher can *distribute* worksheets to students, separating the stack of worksheets into one worksheet per student ("passing out papers"). Often times a teacher needs to do the opposite and "pass in papers", collecting papers from individual students and putting them in one stack. The Distributive Property will let you "pass out" and "pass in" parts.

When you add 0 to any number or multiply any number by 1, the result is *identical* to the original number. The numbers 0 and 1 are called *identities*.

The properties of addition and multiplication are summarized in the chart below.

Property	Operation	Algebra	Numbers	Key Words
Commutative (Change order)	$+$	$a+b=b+a$	$3+4=4+3$	**C**hange **O**rder
Commutative	$\times$	$a\cdot b=b\cdot a$	$5\cdot 2=2\cdot 5$	**C**hange **O**rder
Associative	$+$	$(a+b)+c=a+(b+c)$	$(9+4)+6=9+(4+6)$	Change Grouping
Associative	$\times$	$(a\cdot b)\cdot c=a\cdot(b\cdot c)$	$(9\cdot 5)\cdot 2=9\cdot(5\cdot 2)$	Change Grouping
Identity	$+$	$a+0=a$	$7+0=7$	Add 0
Identity	$\times$	$a\cdot 1=a$	$6\cdot 1=6$	Multiply by 1
Distributive	multiplication over addition	$a(b+c)=a\cdot b+a\cdot c$	$5(23)=5(20)+5(3)$	Distribute Over ( )

The properties allow you to manipulate expressions and compute mentally.

Example: *Simplify* $4\cdot 13\cdot 25$

Using only the Order of Operations to simplify the expression above, you would multiply 4 by 13, then multiply that result by 25. Most students would need pencil & paper. However, using the properties we can solve this problem using mental math.

$(4\cdot 13)25$	Order of Operations, multiply in order from left to right
$=4(13\cdot 25)$	Associative property of multiplication changes grouping
$=4(25\cdot 13)$	Commutative property of multiplication changes order
$=(4\cdot 25)13$	Associative property of multiplication changes grouping
$=100\cdot 13$	Simplify
$=1300$	Simplify

Example: *Simplify* $25\cdot 12$

That could be simplified by just multiplying 25 by 12 using the standard algorithm. Or, we could use the distributive property to break the number apart to perform the multiplications mentally.

$25\cdot 12=25\cdot(10+2)$

$=25(10)+25(2)$

$=250+50$

$=300$

Properties allow us to simplify expressions.

Example: *Simplify the expression* $x + 4 + 2$

$x + 4 + 2$

$= x + (4 + 2)$ Use associative property of addition.

$= x + 6$ Simplify.

Example: *Simplify the expression* $4(8y)$.

$4(8y)$

$= (4 \cdot 8)y$ Use associative property of addition.

$= 32y$ Simplify.

Example: *Write an equivalent expression* $3(x + 7)$.

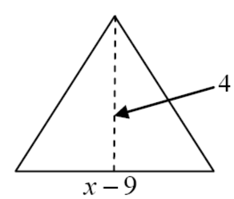
$3(x + 7) = 3x + 3(7)$ Use the distributive property.

$= 3x + 21$ Simplify.

Often standardized tests use geometry to assess knowledge of these properties.

Example: *Find the perimeter of a square whose sides are represented by* $y - 5$ *and simplify the expression. Explain your thinking.*

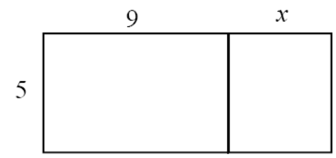
Work	**Explanation**
$P = 4(y - 5)$ $P = 4y - 20$	To find the perimeter, you add the measures of all 4 sides. Since it is a square, I know all 4 sides are equal, so I can multiply the length of one side by 4. I will use the distributive property to simplify.



Example: *Find the area of the triangle and simplify the expression. Show your work.*

Work	**Explanation**
$A = \frac{1}{2}bh$	To find the area, I would use the triangle formula and substitute the values for b and h.
$A = \frac{1}{2}(x - 9)(4)$ $A = \frac{1}{2}(4)(x - 9)$	To simplify the expression, I would use the commutative property to rewrite the order, and then simplify one-half times four.
$A = 2(x - 9)$ $A = 2x - 18$	Then I would use the distributive property.

Example: *Find the area of the rectangle and simplify the expression. Show your work.*



Work	Explanation
$A = bh$	To find the area, I would use the triangle formula and substitute the values for b and h.
$A = (9 + x)(5)$	
$A = (9 \cdot 5) + (x \cdot 5)$	I would use the distributive property, and then simplify the expression.
$A = 45 + 5x$ ***or***	
$A = 5x + 45$	

Simplifying Variable Expressions

A *term* is a number, variable, product or quotient in an expression.

Example: *In the variable expression* $2x + 3y - 5$, *the* $2x, 3y$, *and* -5 *are terms of that expression.*

A *coefficient* is a numerical factor in a term.

Example: *In the variable expression* $2x + 3y - 5$, *the* 2 *is the coefficient of the* x *in the first term; the* 3 *is the coefficient of the* y *in the second term.*

A *constant* term has a number but no variable. It is a quantity that always stays the same.

Example: *In the expression* $2x + 3y - 5$, *the only number without a variable attached is* -5, *so* -5 *is the constant.*

A *variable* term has a number and a variable.

Example: *In the expression* $2x + 3y - 5$, *the* $2x$ *and* $3y$ *are variable terms.*

Like terms are terms that have identical variable parts (i.e., same variable raised to the same power)

Example: *Identify the like terms of the expression* $3x + 4 + 5x$.

$3x$ and $5x$ have identical variable parts, so they are like terms.

Just to review, let's identify the terms, coefficients, and constant terms of the no simplified expression $x + 7 - 4y - 3$ There are four terms: $x, 7, -4y, -3$. The coefficient of x is 1 and the coefficient of y is -4. The constant terms are 7 and -3.

We can write an expression such as $2y + 6y$, as a single term. The distributive property allows us to do this: $2y + 6y = (2 + 6)y = 8y$.

To simplify expressions or combine like terms, group the terms with the same variable raised to the same power. Then add or subtract the coefficients as indicated.

Example:

$\underline{8x^2} + \underline{\underline{9x}} + 7 + \underline{2x^2} - \underline{\underline{4x}} + 3$	Mark like terms**
$(8x^2 + 2x^2) + (9x - 4x) + (7 + 3)$	Group like terms (applying the commutative and associative properties).
$(8 + 2)x^2 + (9 - 4)x + (7 + 3)$	Use the distributive property.
$10x^2 + 5x + 10$	Simplify.

**Another way to mark like terms is to use boxes as shown below. It helps to visualize that the sign in front of the number "belongs" to that number.

$$\boxed{8x^2} + 9x + 7 \boxed{+ 2x^2} - 4x + 3$$

An important link can be made to previous learning. Review place value and expanded notation.

Example:

$$672 = 6(100) + 7(10) + 2(1)$$

$$= 6(10^2) + 7(10) + 2(1)$$

This parallels polynomials $= 6x^2 + 7x + 2$

Now show that you combine like terms in algebra the same way you combine terms in arithmetic.

To add horizontally, from left to right, group the hundreds, the tens and the ones.

Example:

$$241 + 352$$

$= 2(100) + 4(10) + 1(1) + 3(100) + 5(10)$

$= (2 + 3)(100) + (4 + 5)10 + (1 + 2)(1)$

$= 5(100) + 9(10) + 3(1)$

$= \mathbf{593}$

To add polynomials, you add from left to right, grouping terms.

Using the same numerals, an example:

$$(2x^2 + 4x + 1) + (3x^2 + 5x + 2)$$

$= 2x^2 + 4x + 1 + 3x^2 + 5x + 2$

$= (2 + 3)x^2 + (4 + 5)x + (1 + 2)1$

$= \mathbf{5x^2 + 9x + 3}$

Example: *Combine like terms for the expression* $4x + 3 + 5x$.

$4x + 3 + 5x = (4x + 5x) + 3$

$= 9x + 3$

Example: *Simplify* $5(x + 4) - 2x + 5$

Use the distributive property to get rid of the parentheses.

$= 5x + 20 - 2x + 5$

Now combine like terms

$= 5x - 2x + 20 + 5$

$= 3x + 25$

Review - Equations and Their Solutions

An *equation* is a mathematical statement that shows two expressions are equivalent. Another way to define it: an *equation* is a mathematical sentence formed by placing an *equal sign* between two expressions.

A *solution* is a number that produces a true statement when it is substituted for the variable in an equation. (i.e., a solution makes your equation a true statement.)

So, to determine if a value is a solution of an equation simply substitute the given value in place of the variable and simplify to see if this value makes a true statement. If it is true, it is a solution of the equation.

Example: *Given 15 = x – 7, determine whether* $x = 8$ *is a solution.*

$15 = 8 - 7$	Substitute the 8 (value of *x*)
$15 = 1$	Simplify and determine if statement is true or false.
$15 \neq 1$	This is false, so 8 is NOT a solution to this equation.

Note: An equation consists of two expressions connected by an equals sign (=). It can only be true or false. An expression is never true or false, it just has a numerical value.

- Expressions are mathematical phrases whereas equations are complete mathematical statements.
- Equations show relationships whereas expressions don't show any.
- Equations have an equal sign whereas expressions don't have any.
- Equations are to be solved while expressions are to be simplified.
- Equations have a solution while expressions don't have any. *

Students it is also important that students you can write verbal sentences as equations. The following chart provides some verbal expressions that are commonly found in algebra. Please note the ** indicates particularly difficult ones to understand.

*Read more: *Difference Between Expression and Equation | Difference Between* http://www.differencebetween.net/miscellaneous/difference-between-expression-and-equation/#ixzz4ffUMuusU

Operation	Verbal Expression	Algebraic Expression
Addition +	a number plus 7	$n + 7$
Addition +	8 added to a number	$n + 8$
Addition +	a number increased by 4	$n + 4$
Addition +	5 more than a number	$n + 5$
Addition +	the sum of a number and 6	$n + 6$
Addition +	Tom's age 3 years from now	$n + 3$
Addition +	two consecutive integers	$n, n+1$
Addition +	two consecutive odd integers	Let $x = 1st$ odd, $x+2 = 2nd$ odd
Addition +	2 consecutive even integers	Let $x = 1st$ even, $x+2 = 2nd$ even
Subtraction –	a number minus 7	$x - 7$
Subtraction –	8 subtracted from a number**	$x - 8$
Subtraction –	a number decreased by 4	$x - 4$
Subtraction –	4 decreased by a number	$4 - x$
Subtraction –	5 less than a number**	$x - 5$
Subtraction –	the difference of a number and 6	$x - 6$
Subtraction –	Tom's age 3 years ago	$x - 3$
Subtraction –	separate 15 into two parts**	$x, 15 - x$
Multiplication • ( )	12 multiplied by a number	$12n$
Multiplication • ( )	9 times a number	$9n$
Multiplication • ( )	the product of a number and 5	$5n$
Multiplication • ( )	Distance traveled in *x* hours at 50 mph	$50x$
Multiplication • ( )	twice a number	$2n$
Multiplication • ( )	half of a number	$\frac{1}{2}n$
Multiplication • ( )	number of cents in *x* quarters	$25x$
Division ÷	a number divided by 12	$\frac{x}{12}$
Division ÷	the quotient of a number and 5	$\frac{x}{5}$
Division ÷	8 divided into a number**	$\frac{x}{8}$

Examples: *Write the verbal sentence as an equation. Solve using mental math.*

a) The sum of a number and 5 is 12. $n + 5 = 12,\ n = 7$

b) The product of a number and 4 is 20. $4x = 20,\ x = 5$

c) 12 equals what number plus 2? $12 = a + 2,\ a = 10$

d) 7 is the quotient of what number divided by 5? $7 = \frac{y}{5},\ y = 35$

Two other commonly used words are to "*square*" or "*cube*" a number. They are represented as $x^2,\ x^3$.

Review - Solving Equations

To *solve an equation* is to find the value(s) of *x* which make the equation a true statement.

Strategy for Solving Equations: To solve linear equations, put the variable terms on one side of the equal sign, and put the constant (number) terms on the other side. To do this, use opposite or inverse operations. *Inverse operations* are two operations that undo each other, such as addition and subtraction. There are several properties that will help us obtain equivalent equations by performing inverse operations.

Properties of Equality

Addition Property of Equality	if $a = b$, then $a + c = b + c$
Subtraction Property of Equality	if $a = b$, then $a - c = b - c$
Multiplication Property of Equality	if $a = b$, then $ac = bc$
Division Property of Equality	if $a = b, c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$

We used the Order of Operations to evaluate expressions such as $5 + 3x$ when $x = 4$.

The expression $5 + 3x$ was rewritten substituting 4 for x.

$$\begin{aligned} &5x + 3x \\ &= 5 + 3(4) \\ &= 5 + 12 \\ &= 17 \end{aligned}$$

So, we now know that $5 + 3x = 17$ when $x = 4$.

The question becomes, can we find the value of *x*, the solution, if we know that $5 + 3x = 17$?

To undo a variable expression and isolate the variable, we must use the Order of Operations in reverse using the opposite (inverse) operations. Let's look at a gift-wrapping analogy to better understand this strategy. When a present is wrapped, it is placed in a box, the cover is put on, the box is wrapped in paper, and finally a ribbon is added to complete the project. To get the present out of the box, everything would be done in reverse order, performing the opposite, INVERSE OPERATION. First, we take off the ribbon, then take off the paper, next take the cover off, and finally take the present out of the box.

To isolate the variable means to have all the variables on one side of an equation and the numbers on the other side. In this chapter, we will practice one-step equations.

Example: *Solve* $x + 5 = 9$

To solve this equation, undo the variable expression on the left side by using the order of operations in reverse using the inverse operation.

We have addition and to get rid of that we subtract 5 from both sides. This is applying the Subtraction Property of Equality. Check your answer!

$$x + 5 = 9$$

$$x + 5 - 5 = 9 - 5$$

$$x = 4$$

$$4 + 5 = 9 \quad \checkmark$$

It is also common practice to simply show subtracting five from both sides as shown below.

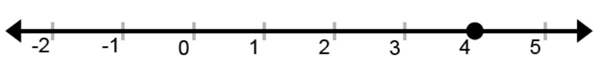
$$x + 5 = 9$$

$$-5 = -5$$

$$x = 4$$

$$4 + 5 = 9 \quad \checkmark$$

We can also introduce what the graph of the solution would look like at this point.



Example: *Find the solution of.* $y - 3 = 12$

There is a subtraction of 3; to get rid of that, add 3 to both sides (Addition Property of Equality).

$$y - 3 = 12$$
$$y - 3 + 3 = 12 + 3$$
$$y = 15$$
$$15 - 3 = 12 \quad ✓$$

or

$$y - 3 = 12$$
$$+3 = +3$$
$$y = 15$$
$$15 - 3 = 12 \quad ✓$$

Example: *Solve the equation* $-4x = 32$.

There are no additions or subtractions. To isolate the variable, undo the multiplication by dividing both sides of the equation by –4.

$$-4x = 32$$
$$\frac{-4x}{-4} = \frac{32}{-4}$$
$$x = -8$$
$$-4(-8) = 32 \quad ✓$$

Example: *Solve for y.* $\frac{y}{3} = 10$

There are no additions or subtractions, so we undo the division by multiplying both sides by 3.

$$\frac{y}{3} = 10$$
$$3\left(\frac{y}{3}\right) = 3(10)$$
$$y = 30$$
$$\frac{30}{3} ✓ 10$$

The rules of math do not change when using different number sets. So we solve equations the same whether there are fractions, decimals, or integers in the equation. This is an excellent time to review operations with decimals by solving one-step equations with decimals.

Example:

$$x + 4.25 = -2.5$$
$$x + 4.25 - 4.25 = -2.5 - 4.5$$
$$x = -6.75$$
$$-6.75 + 4.25 = -2.5 \quad ✓$$

or

$$x + 4.25 = -2.5$$
$$-4.25 = -4.5$$
$$x = -6.75$$
$$-6.75 + 4.25 = -2.5 \quad ✓$$

Review - Solving Two-Step Equations

The general strategy for solving a multi-step equation in one variable is to rewrite the equation in $ax + b = c$ format (where *a*, *b*, and *c* are real numbers), then solve the equation by isolating the variable using the Order of Operations in reverse and using the opposite operation. (Remember the analogy to unwrapping a gift…)

↑

<u>Order of Operations</u>

1. Parentheses (Grouping)
2. Exponents
3. Multiply/Divide, left to right
4. Add/Subtract, left to right

Evaluating an arithmetic expression using the Order of Operations will suggest how we might go about solving equations in the $ax + b = c$ format.

To evaluate an arithmetic expression such as $4 + 2 \cdot 5$, we'd use the Order of Operations.

$$\begin{aligned} 4 + \underline{2 \cdot 5} &= \quad \text{First we multiply, } 2 \cdot 5 \\ 4 + 10 &= \quad \text{Second we add, } 4 + 10 \\ 14 & \end{aligned}$$

Now, rewriting that expression, we have $2 \cdot 5 + 4 = 14$, a form that leads to equations written in the form $ax + b = c$.

If I replace 5 with *n*, I have

$$\begin{aligned} &2 \cdot n + 4 = 14 \ or \\ &2n + 4 = 14, \\ &\text{an equation in the } ax + b = c \text{ format.} \end{aligned}$$

To solve that equation, I am going to "undo" the expression "$2n + 4$". *I will isolate the variable by using the Order of Operations in reverse and using the inverse operation.*

That translates to getting rid of any addition or subtraction first, then getting rid of any multiplication or division next. Undoing the expression and isolating the variable results in finding the value of *n*.

This is what it looks like:

$$\begin{aligned} 2n + 4 &= 14 \\ 2n + 4 - 4 &= 14 - 4 \quad \text{subtract 4 from each side to "undo" the addition} \\ 2n &= 10 \\ \frac{2n}{2} &= \frac{10}{2} \quad \text{divide by 2 to "undo" the mulitplication} \\ n &= 5 \end{aligned}$$

$$\begin{aligned} 2n + 4 &= \ 14 \\ -4 &= -4 \\ 2n &= 10 \\ \frac{2n}{2} &= \frac{10}{2} \\ n &= 5 \end{aligned}$$

Check your solution by substituting the answer back into the original equation.

$$\begin{aligned} 2n+4&=14 && \text{original equation} \\ 2(5)+4&=14 && \text{substitute '5' for '}n\text{'} \\ 10+4&=14 && \\ 14&=14 && \text{true statement, so my solution is correct} \end{aligned}$$

Example: *Solve for x,* $3x-4=17$.

Using the general strategy, we always want to "undo" whatever has been done in reverse order. We will undo the subtracting first by adding, and then undo the multiplication by dividing.

$$\begin{array}{ccc} 3x-4=17 & \quad or \quad & 3x-4=17 \\ 3x-4+4=17+4 & & +4=+4 \\ 3x=21 & & 3x=21 \\ \dfrac{3x}{3}=\dfrac{21}{3} & & \dfrac{3x}{3}=\dfrac{21}{3} \\ x=7 & & x=7 \end{array}$$

Check:

$$\begin{aligned} 3x-4&=17 \\ 3(7)-4&=17 \\ 21-4&=17 \\ 17&=17\checkmark \end{aligned}$$

Example: *Solve for x,* $\frac{x}{4}+5=12$

$$\begin{array}{ccc} \dfrac{x}{4}+5=12 & \quad or \quad & \dfrac{x}{4}+5=12 \\ \dfrac{x}{4}+5-5=12-5 & & -5=-5 \\ \dfrac{x}{4}=7 & & \dfrac{x}{4}=7 \\ (4)\left(\dfrac{x}{4}\right)=(4)(7) & & (4)\left(\dfrac{x}{4}\right)=(4)(7) \\ x=28 & & x=28 \end{array}$$

Check:

$$\begin{aligned} \frac{x}{4}+5&=12 \\ \frac{(28)}{4}+5&=12 \\ 7+5&=12 \\ 12&=12\checkmark \end{aligned}$$

***NOTE:** Knowing how to solve equations in the $ax + b = c$ format is extremely important for success in algebra. All other equations will be solved by converting equations to $ax + b = c$. To solve systems of equations, we rewrite the equations into one equation in the form $ax + b = c$ and solve.

Review - Solving Equations that Are NOT in the $ax + b = c$ Format

The general strategy for solving equations NOT in the $ax + b = c$ format is to rewrite the equation in $ax + b = c$ format using the Properties of Real Numbers.

An important problem solving and learning strategy is to take a problem that you don't know how to do and transform that into a problem that you know how to solve. Right now, we know how to solve problems such as $ax + b = c$. I cannot make that problem more difficult, but what I can do is make it look different.

For instance, if I asked you to solve $5(2x + 3) - 4 = 21$, that problem looks different from the $2x + 4 = 14$ that we just solved by using the Order of Operations in reverse. The question you must ask is, "What is physically different in these two problems?"

The answer is the new problem has parentheses. That means to make these problems look alike, we need to get rid of those parentheses. We'll do that by using the distributive property.

$$\begin{aligned} 5(2x+3)-4&=21 && \\ 10x+15-4&=21 && \text{we applied the distributive property} \\ 10x+11&=21 && \text{we combined like terms} \end{aligned}$$

Now I have converted $5(2x + 3) - 4 = 21$ into $ax + b = c$ format as $10x + 11 = 21$. We recognize this format and know how to solve it.

$$\begin{aligned} 10x+11&=21 \\ 10x+11-11&=21-11 \\ 10x&=10 \\ \frac{10x}{10}&=\frac{10}{10} \\ x&=1 \end{aligned} \qquad or \qquad \begin{aligned} 10x+11&=\ \ 21 \\ -11&=-11 \\ 10x&=10 \\ \frac{10x}{10}&=\frac{10}{10} \\ x&=1 \end{aligned}$$

Check:

$$\begin{aligned} &5(2x+3)-4=21 \\ &5[2(1)+3]-4=21 \\ &5[2+3]-4=21 \\ &5[5]-4=21 \\ &25-4=21 \\ &21=21\checkmark \end{aligned}$$

Example: *Solve for x,* $4(3x-2)+5=45$

$$
\begin{aligned}
4(3x-2)+5&=45\\
12x-8+5&=45\\
12x-3&=45\\
12x-3+3&=45+3\\
12x&=48\\
\frac{12x}{12}&=\frac{48}{12}\\
x&=4
\end{aligned}
$$

The parentheses make this problem look different than the $ax + b = c$ problems.

Get rid of the parentheses by using the distributive property, and then combine terms.

Now it is up to you to check the answer!

Caution! Note that when we solved these equations we got rid of the parentheses first. The reason we get rid of the parentheses first is because the strategy is to rewrite the equation in $ax+b=c$ format before using the Order of Operations in reverse.

Solving Linear Equations with Variables on Both Sides

Using the same strategy, we rewrite the equation so it is in $ax+b=c$ format.

Example: *Solve for x,* $5x+2=3x+14$.

We ask ourselves, "What is physically different in this problem?" This equation is different because there are variables on both sides of the equation. The strategy is to rewrite the equation in $ax+b=c$ format which calls for variables on only one side of the equation.

What's different? There is a $3x$ on the other side of the equation. How do we get rid of the addition of $3x$?

$5x + 2 = 3x + 14$	
$5x + 2 - \mathbf{3x} = 3x + 14 - \mathbf{3x}$	Subtraction Property of Equality
$5x - \mathbf{3x} + 2 = 3x - \mathbf{3x} + 14$	Commutative Property of Addition
$2x + 2 = 14$	Combining like terms

Now the equation is in $ax + b = c$ format.

$$2x + 2 = 14$$

$2x + 2 - 2 = 14 - 2$ Subtraction Property of Equality

$2x = 12$ Arithmetic fact

$\frac{2x}{2} = \frac{12}{2}$ Division Property of Equality

$x = 6$ Arithmetic fact

Let's try a few more examples:

$5x - 17 = 3x - 9$	$-2x - 17 = 6 - x$	$3m + 8 - 5m = 9 + 4m + 29$
$-3x \quad\quad -3x$	$+x \quad\quad +x$	$-2m + 8 = 4m + 38$
$2x - 17 = -9$	$-1x - 17 = 6$	$-2m - 4m + 8 = 4m - 4m + 38$
$+17 \quad +17$	$+17 \quad +17$	$-6m + 8 = 38$
$2x = 8$	$-1x = 23$	$-8 \quad -8$
$\frac{2x}{2} = \frac{8}{2}$	$\frac{-1x}{-1} = \frac{23}{-1}$	$-6m = 30$
$x = 4$	$x = -23$	$\frac{-6m}{-6} = \frac{30}{-6}$
		$m = -5$

Now, we can make equations longer, but I cannot make them more difficult!

Example: *Solve for x,* $5(x + 2) - 3 = 3(x - 1) - 2x$.

In this problem, there are parentheses and variables on both sides of the equation. It is clearly a longer problem. The strategy remains the same: rewrite the equation in $ax + b = c$ format, and then isolate the variable by using the Order of Operations in reverse using the opposite operation.

Physically, this equation looks different because there are parentheses, so let's get rid of them by using the distributive property.

$5(x+2) - 3 = 3(x-1) - 2x$	
$5x + 10 - 3 = 3x - 3 - 2x$	Distributive Property
$5x + 7 = x - 3$	Combine like terms
$5x + 7 - x = x - 3 - x$	Subtraction Property of Equality
$4x + 7 = -3$	Combine like terms
$4x + 7 - 7 = -3 - 7$	Addition Property of Equality
$4x = -10$	Simplify
$\frac{4x}{4} = \frac{-10}{4}$	Division Property of Equality
$x = \frac{-5}{2} or -2.5$	Simplify

Caution! When solving equations, we often write arithmetic expressions such as $-3 + (-7)$ as $-3 - 7$ as we did in the above problem. We need to remember that when there is <u>not</u> an "extra" minus sign, the computation is understood to be an addition problem.

Examples:

$(+5) + (+6) = 5 + 6 = 11$

$(-5) + (-6) = -5 - 6 = -11$

$(-8) + (+3) = -8 + 3 = -5$

Also, make sure you understand how to simplify expressions with several negatives.

Examples:

$5 - (-4) = 5 + 4 = 9$

$-3 - (-8) = -3 + 8 = 5$

$-2 - 5 = -2 + (-5) = -7$

Check Yourself: Solve Equations with Variables on Both Sides

1. $8x - 14 = 6x + 16$
2. $12x - 14 = 6x + 10$
3. $3(3y - 6) = 7y - 18 + 2y$
4. $0.24x + 2(.03 + 3) = 12$

Solving Equations with "No Solutions" or "Infinitely Many" Solutions

When you solve an equation, you *may* find something unusual happens. For instance, when you subtract the variable from both sides (in an effort to get the variable term on one side of the equation), no variable term remains! Your solution is either "no solution" or "infinitely many". Look at the following examples.

Example: *Solve* $3(3x - 1) = 9x$.

$3(3x - 1) = 9x$

$9x - 3 = 9x$ Notice we could stop now if we recognized that it is impossible for a number $9x$ to be equal to 3 less than itself. If we continue….

$$9x - 3 = 9x$$
$$-9x \qquad -9x$$
$$-3 = 0$$

…we still have a statement that is not true.

So, the equation is said to have NO SOLUTION.

Example: *Solve* $8x - 2 = 2(4x - 1)$.

$$8x - 2 = 2(4x - 1)$$
$$8x - 2 = 8x - 2$$

Notice that this statement would be true no matter what value we substitute for x. We could go further….

$$8x - 2 = 8x - 2$$
$$-8x = -8x$$
$$-2 = -2$$

…we still get a true statement.

So, the equation is said to have a solution of INFINITELY MANY.

Check Yourself: Determine Whether the Equation has No Solution, One Solution, or Infinite Many Solution by Placing a Check in the Appropriate Column.

Equation	No Solution	One Solution	Infinite Solutions
$2(3x + 4) = 6x + 8$			
$2x - 5 = 5$			
$3x - 1 = -3x - 1$			
$-2x + 11 = -2x - 11$			

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- SOLVE LINEAR EQUATIONS WITH INTEGERS BY USING INVERSE OPERATIONS
- SOLVE LINEAR EQUATIONS WITH INTEGERS USING THE DISTRIBUTIVE PROPERTY

TAKE THE **MAFS.8.EE.3.7** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.EE.3.7**

I SCORED BELOW 70% ON THE EDGENUITY

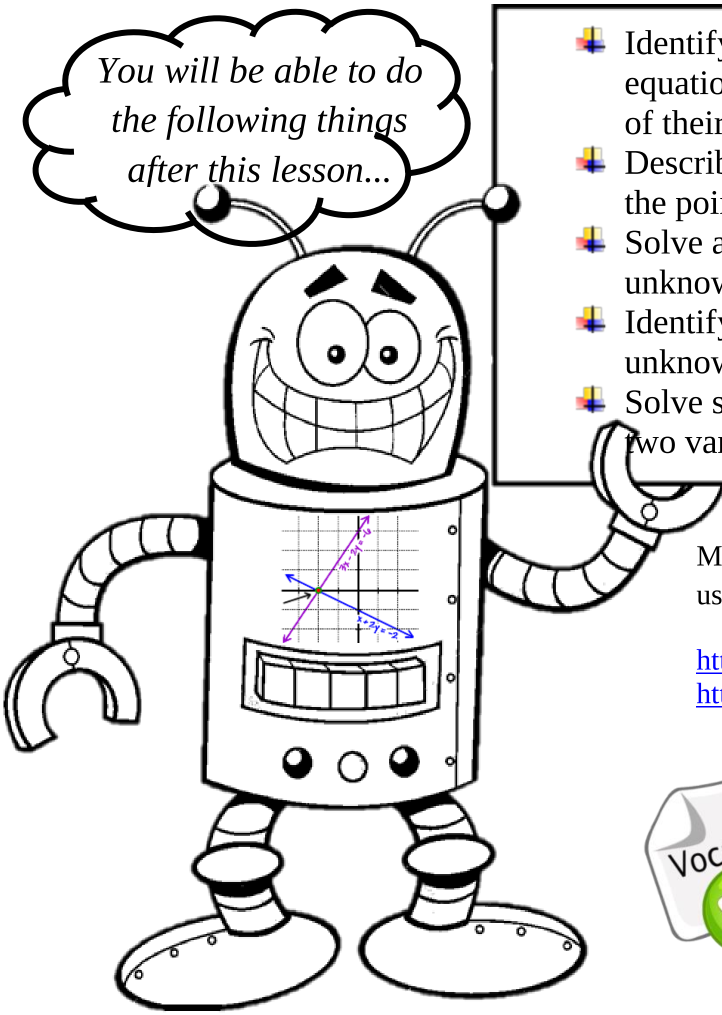
- COMPLETE LESSON **MAFS.8.EE.3.7** ON EDGENUITY
- GO TO LESSON 12: ANALYE AND SOLVE PAIRS OF SIMULTANEOUS LINEAR EQUATIONS

I SCORED 70% OR ABOVE ON THE EDGENUITY

- GO TO LESSON 12: ANALYZE AND SOLVE PAIRS OF SIMULTANEOUS LINEAR EQUATIONS

The resources in this packet have been compiled from the Regional Professional Development Program and Mr. Bright

Lesson 12: Analyze and Solve Pairs of Simultaneous Linear Equations



- Identify the solution(s) to a system of two linear equations in two variables as the point(s) of intersection of their graphs
- Describe the point(s) of intersection between two lines as the points that satisfy both equations simultaneously
- Solve a system of two equations (linear) in two unknowns algebraically
- Identify cases in which a system of two equations in two unknowns has no solution.w
- Solve simple cases of systems of two linear equations in two variables by inspection

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- System of linear equations
- Simultaneous Equations
- Linear Equation
- Parallel
- Coincident
- Substitution
- Elimination
- Inspection

Solving Systems Graphically

Now that we know how to solve complicated equations, we move on to solving what are called systems of equations. A system of equations is when we have multiple equations with multiple variables and we are looking for values that the variables represent so that all of the equations are true at the same time.

We will mainly be dealing with two variables and two equations, but you can solve most systems of equations as long as you have the same number of equations as variables. As a quick example, consider the following system:

$$x + y = 5$$

$$x - y = 1$$

It doesn't take too much work to verify the solution of this system is x = 3 and y = 2. Notice that those values for x and y make both equations true at the same time.

$$3 + 2 = 5$$

$$3 - 2 = 1$$

The question remains, how do we get that solution?

Solving with Graphs

If we have our equations set up using the x and y variables, we can graph both equations. Let's see how this helps us. To start with, let's graph the first equation $x + y = 5$. Remember that we can do this in a couple of ways. We could simply make an x/y chart and plot the points. Alternately, we could get the equation in slope- intercept form and then graph.

Let's start with an x/y chart. Remember that in an x/y chart we pick x values and substitute those into the equation to find y values. Confirm on your own that this x/y chart is correct for $x + y = 5$:

x	-2	-1	0	1	2
y	7	6	5	4	3

Now we can plot those points on a coordinate plane and connect them to get our graph.

If we don't like the x/y chart method, we can turn the equation into slope-intercept form by isolating the y variable on the left side like so:

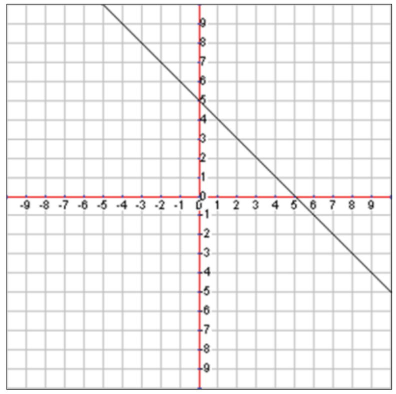
$x + y = 5$

$x - \boldsymbol{x} + y = 5 - \boldsymbol{x}$ ← Subtract x from both sides

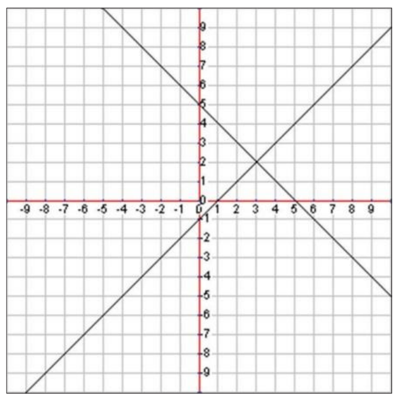
$y = 5 + (-x)$ ← Subtract means add a negative

$y = -x + 5$ ← Commutative property

Either way, we'll get a graph that looks like this:



Now we graph the second equation, x – y = 1, in the same way. It turns into y = x – 1 and gives us an overall graph like the following:



What do you notice about those two lines? They intersect. At what point do they intersect? The intersection is at the point (3, 2) which means that x = 3 and y = 2. What does this tell us about solving systems of equations using graphs?

Yes, the point of intersection is the solution to the system because that point is the only point on both lines (assuming we're dealing with only linear equations for now). In fact, we sometimes write the solution to a system of equations as a point. So, the solution to this system is (3, 2).

Let's try another example. What is the solution to the following system of equations?

$$4x + 2y = 6$$
$$-x + 2y = -4$$

Let’s work it out!

$$4x + 2y = 6 \qquad\qquad -x + 2y = -4$$

$$2y = -4x + 6 \qquad\qquad 2y = x - 4$$

$$y = \frac{-4x + 6}{2} \qquad\qquad y = \frac{x - 4}{2}$$

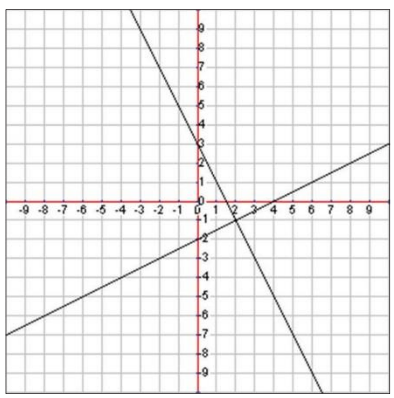
$$y = -2x + 3 \qquad\qquad y = \frac{1}{2}x - 2$$

We'll leave it as an exercise to verify that the following equations are the same system just written in slope-intercept form:

$$y = -2x + 3$$

$$y = \frac{1}{2}x - 2$$

Now graph those equations to see where they intersect.



It looks like the graphs intersect at the point (2, -1) which we can verify by substituting into the original equations as follows:

$$4x + 2y = 6 \rightarrow 4(2) + 2(-1) = 6$$

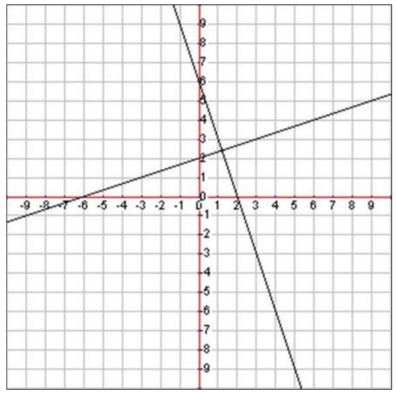
$$-x + 2y = -4 \rightarrow -(2) + 2(-1) = -4$$

That means we have the correct solution.

Estimating Using a Graph

So far, our solutions have been integer values, but that won't always be the case. We can still use the graphing method to get a decent estimate even if it's not a very nice solution. For example, consider the following equations and graphs.

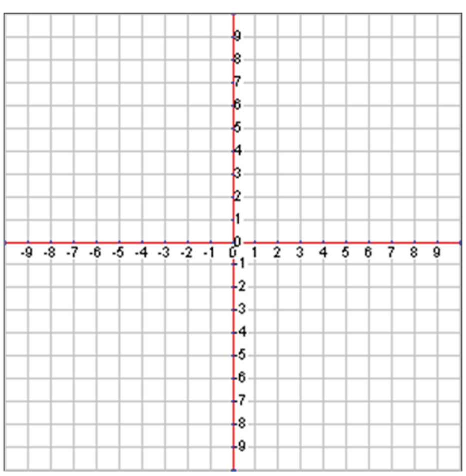
$$y = \frac{1}{3}x - 2$$
$$y = -3x + 6$$



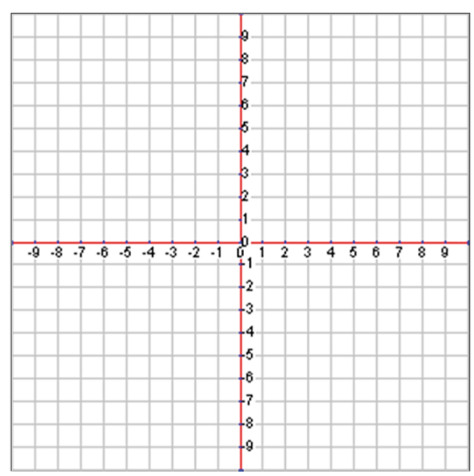
Note that the x-coordinate where the lines intersect is a little more than 1 and the y-coordinate of intersection is little more than 2. We might estimate this solution as ($1\frac{1}{4}$, $2\frac{1}{3}$) or the decimal equivalent. The actual solution is (1.2, 2.4) for this system, but we'll discover how to find the exact solution later.

Check Yourself: Graph the following systems of equations and estimate the solution from the graph.

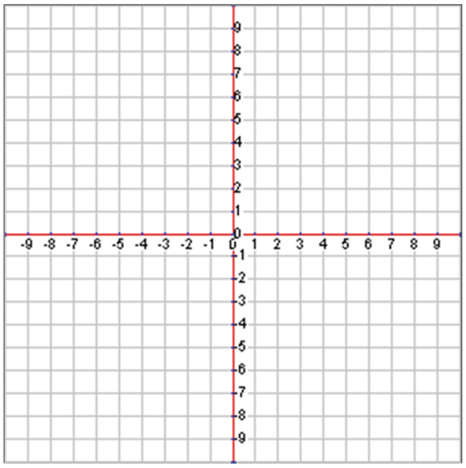
1. $y = 2x + 8$
   $y = x + 6$



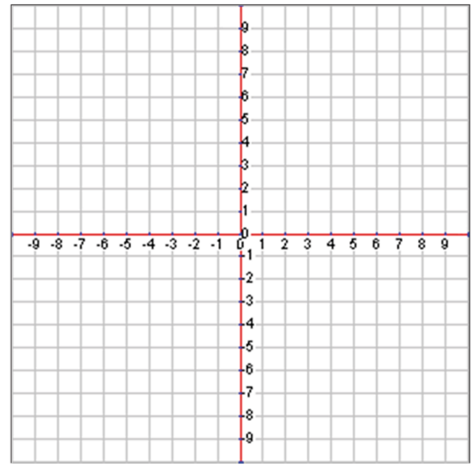
2. $y = -2x + 3$
   $y = \frac{1}{2}x - 4$



3. $3y = x + 9$
$2y = -4x - 8$



4. $4x + 2y = 6$
$-6x + 2y = 6$



Solving Systems with Substitution

While graphing is useful for an estimate, the main way that we can solve a system to get an exact answer is algebraically. There are a few useful methods to do this, and we will begin with the substitution method. The general idea with this method is to isolate a single variable in one equation and substitute that into the other equation.

Isolating a Variable

Consider the following system of equations.

$$3x + y = 1$$

$$3x + 2y = 4$$

It is always best to check if one variable has a coefficient of one and isolate that variable. Remember that a coefficient is a number multiplied by a variable. That means that a coefficient of one will mean that the variable doesn't have a number in front of it because the one is understood to be there, and we don't write it. In this case, notice that the y in the first equation has a coefficient of one. It would probably be easiest to isolate that variable. Let's do so.

$$3x - 3x + y = 1 - 3x$$
$$y = 1 - 3x$$

Substitution

Now that we know what *y* is equal to in the first equation, we can substitute that expression for *y* in the second equation. Be careful to not plug back into the first equation or else we'll end up with infinite solutions every time. Since we want a solution that is true in both equations, we must use both equations.

$$3x + 2y = 4$$

$$3x + 2(1 - 3x) = 4$$

$$3x + 2 - 6x = 4$$

$$-3x + 2 = 4$$

Now that we have it down to a simple two-step equation, we can solve like normal and get the following:

$$-3x + 2 = 4$$

$$3x + 2 - 2 = 4 - 2$$

$$\frac{-3x}{-3} = \frac{2}{-3}$$

$$x = -\frac{2}{3}$$

Finding the Second Variable Value

Now that we know what *x* equals, we can substitute that back into either of the original equations to find what the *y* coordinate is at the point of intersection of the two lines. It is also a good idea to plug in this *x* value into both equations to make sure they give the same *y* value. We'll start with the first equation.

$$3x + y = 1$$

$$3\left(-\frac{2}{3}\right) + y = 1$$

$$-2 + y = 1$$

$$-2 + 2 + y = 1 + 2$$

$$y = 3$$

Double Check

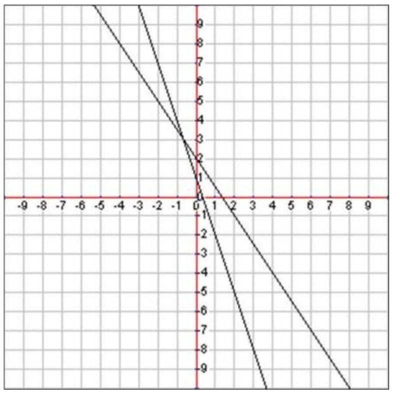
This means that the solution should be the point $\left(-\frac{2}{3}, 3\right)$, but we found that *y* value using the first equation. We need to make sure this point is on the second line as well, so let's substitute the values into that equation.

$$3x + 2y = 4$$

$$3\left(-\frac{2}{3}\right) + 2(3) = 4$$

$$-2 + 6 = 4$$

That statement is true and therefore the point is on the second line as well. So, our solution is the point $\left(-\frac{2}{3}, 3\right)$ for this system. Just for some extra confidence, examine the following graph of the system of equations and notice that the point we found is indeed the point of intersection.



Coefficients Other Than One

It may be the case that we have all coefficients with values other than one. We can still use substitution, but we'll have to be a bit more careful isolating one variable at the beginning. Let's consider the following system of equations.

$$2x + 4y = 8$$

$$3x + 2y = 7$$

In this case, it might be easier to solve the first equation for *x* because the coefficient for *y* and the 8 will easily divide by 2. So, let's isolate the *x* in the first equation as follows:

$$2x + 4y - 4y = 8 - 4y$$
$$2x = 8 - 4y$$
$$x = \frac{8 - 4y}{2}$$
$$x = 4 - 2y$$

Now substitute that *x* value into the second equation as follows:

$$3x + 2y = 7$$
$$3(4 - 2y) + 2y = 7$$
$$12 - 6y + 2y = 7$$
$$12 - 4y = 7$$
$$12 - 12 - 4y = 7 - 12$$
$$-4y = -5$$
$$y = \frac{-5}{-4}$$
$$y = \frac{5}{4}$$

Now that we have the *y* coordinate, we can plug in to find the *x* value.

$$2x + 4y = 8$$
$$2x + 4\left(\frac{5}{4}\right) = 8$$
$$2x + 5 = 8$$
$$2x + 5 - 5 = 8 - 5$$
$$2x = 3$$
$$\frac{2x}{2} = \frac{3}{2}$$
$$x = \frac{3}{2}$$

So, our solution is $\left(\frac{3}{2}, \frac{5}{4}\right)$. We'll leave it as an exercise to double check using the second equation.

Infinite and No Solutions

It is still possible to get infinite solutions or no solution for a system of equations. After the substitution step, if we get down to a number equals a number statement that is always true, there are infinite solutions. If we get down to a number equals a number statement that is false, there are no solutions. This is really the application of what we learned earlier in this unit about solving equations with one variable and getting infinite or no solutions.

Check Yourself: Solve the following systems using the substitution method.

1. $2x + 8y = 12$
   $x - 2y = 0$

2. $x + y = 7$
   $2x + y = 5$

3. $y = -\frac{1}{2} + 1$
   $2x + 3y = 6$

4. $2x - \frac{1}{3}y = -9$
   $-3x + y = 15$

Solving Systems with Elimination

Sometimes it is easier to eliminate a variable entirely from a system of equations rather than use the substitution method. We do this by adding opposite coefficients together to get zero of one variable.

Subtracting to Eliminate

We first need to decide which variable is easiest to eliminate. Consider the following system of equations.

$$3x + y = 1$$

$$3x + 2y = 4$$

Notice that in this case the coefficients for x are the same. This means that they will be easily eliminated. If we subtract $3x$ from both sides of the first equation, we will eliminate the variable from the left side but will still have x are the right side. However, if we instead subtracted $(3x + 2y)$ from the left side we could subtract 4 from the right side because we know that $3x + 2y$ is exactly equal to 4 thanks to the second equation. (Remember that if we're going to solve a system of equations, we'll have to use both equations somehow, which is what we just did.)

This is sort of like repossession in a way. If you don't have the money to pay the bank, they can repossess your property up to an equivalent value of what you owe. In the same way, if we don't want to take away $(3x + 2y)$ from the right side, we can take away something equivalent which is 4 in this case. Let's take a look.

$$\begin{array}{r} 3x + y = 1 \\ \underline{-(3x + 2y = 4)} \\ 0x - 1y = -3 \end{array}$$

Notice that it almost looks like we subtracted the second equation from the first. What we actually did, was subtract expressions that are equal from both sides to keep the first equation balanced. Now we can solve since we have zero *x*'s left.

$$-1y = -3$$

$$y = \frac{-3}{-1}$$

$$y = 3$$

Now that we know what *y* equals, we can substitute that back into either equation to find the *x* value of the solution point.

$$3x + y = 1$$

$$3x + 3 = 1$$

$$3x + 3 - 3 = 1 - 3$$

$$3x = -2$$

So, we get the solution $\left(-\frac{2}{3}, 3\right)$ which you can verify is in the second equation by substituting both values in to make sure it is a true mathematical statement.

Adding to Eliminate

Adding to eliminate a variable will work the same way. In this case we should find one variable with the opposite coefficient of the same variable in the other equation. For example, consider this system of equations:

$$3x + 2y = 4$$

$$x - 2y = 4$$

Notice that the first equation has 2 as the coefficient for *y* and the second equation has a -2 as the coefficient. That means we should be able to add ($x - 2y$) to both sides of the first equation. However, remember that we don't want to end up with more of the *y* variable on the right side, so we will add something equivalent to it. In this case that will be 4.

$$\begin{array}{r} 3x + 2y = 4 \\ \underline{+(x \quad - \ 2y = 4} \\ 4x + 0y = 8 \end{array}$$

We get 0y which means that the y variable is eliminated

$$4x = 8$$

$$x = \frac{8}{4}$$

$$x = 2$$

We'll leave it as an exercise to show that from here we can get $y = -1$ which means that our solution to this system of equations is the point $(2, -1)$.

When the Coefficients Don't Match

The elimination method works fine when the coefficients match or are opposites, but what about when it is just a messy system of equations like this?

$$2x + 3y = -1$$

$$4x + 5y = -1$$

Solving this system by the substitution method would mean dealing with fractions and the coefficients don't match so it looks like the elimination method won't work either. However, is there a way we can get the coefficients to match?

Notice that in the first equation we have a $2x$ and in the second we have a $4x$. Wouldn't it be nice if the first equation had a $4x$ instead of the $2x$? Is there any way we can make that happen? If we multiply both sides of the first equation by 2, we will maintain equality and have $4x$ to match the second equation. Let's do so.

$$2(2x + 3y = -1)$$

$$4x + 6y = -2$$

Now that we have matching coefficients we can use the elimination method to continue to solve by subtracting $(4x + 6y)$ from the left side of our new equation and subtracting -2 from the right side since that is equal to $4x + 6y$.

$$\begin{array}{r} 4x + 6y = -2 \\ \underline{-(4x + 5y = -1)} \\ 0x + 1y = -1 \end{array}$$

$$y = -1$$

From here we can again substitute $y = -1$ into either original equation to find that $x = 1$ which gives us the solution of $(1, -1)$.

Infinite and No Solutions

It is still possible to get infinite solutions or no solution for a system of equations. If both variables get eliminated and we get down to a number equals a number statement that is always true, there are infinite solutions. If we get down to a number equals a number statement that is false, there are no solutions.

Check Yourself: Solve the following systems using the elimination method.

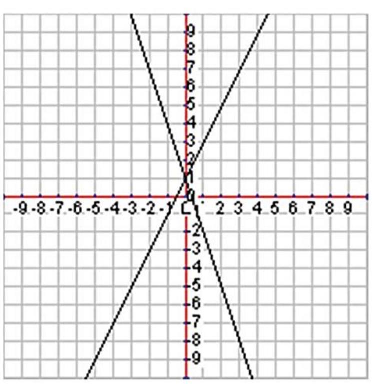
1. $x + y = 1$
   $x - y = 5$

2. $\frac{1}{2}x + 2y = -10$
   $2x + 2y = -10$

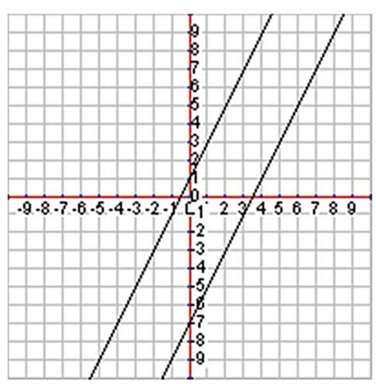
3. $3x + y = 2$
   $6x + 3y = 5$

Solving Systems by Inspection

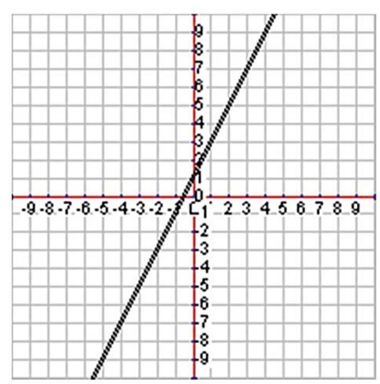
What makes a system of linear equations have a single solution, no solutions, or infinite solutions? One of the first representations we looked at for systems was the graphical representation. What is true about the following systems of linear equations that have either infinite or no solution?



One solution
$y = 2x + 1$
$y = -3x + 1$



No solutions
$y = 2x + 1$
$y = 2x - 7$



Infinite solutions
$y = 2x + 1$
$y = 2x + 1$

Notice that the systems with no solutions and infinite solutions both have the same slope.

In other words, the lines are parallel. If those parallel lines have different *y*-intercepts, then there are no solutions to the system. If those parallel lines are in fact the same exact line (same slope and same *y*-intercept), then there are infinite solutions. The lines are sitting right on top of each other. Therefore, if we could quickly determine whether two lines have the same slope, we could know if it will have infinite or no solutions.

Standard Form

If the two equations are given in slope-intercept form, then we can readily see the slope and *y*-intercept. Same slope and different intercept would mean no solution. Same slope and same intercept would mean infinite solutions. However, not all equations are given in slope-intercept form. Another common form of a linear equation is called **standard form**, which is: A*x* + B*y* = C.

Consider the following system of equations given in standard form. We can't readily see the slope or *y*- intercept since they are both in standard form.

$$2x + y = 5$$

$$4x + 2y = 10$$

So how can we find the slope? We could solve each equation for *y*, but this method is called inspection. We're looking for a quicker way. Let's get the second equation in slope-intercept form and see if we can find any patterns of where the slope comes from.

$$4x + 2y = 10$$

$$4x - 4x + 2 = 10 - 4x$$

$$2y = -4x + 10$$

$$2y{-}4x + 10$$

$$y = \text{-}2x + 5$$

Notice that we got the slope from dividing the coefficients of the variables. Specifically, if we started with the standard form equation A*x* + B*y* = C, we took –A divided by B. In other words, we can simply look at the ratio of the coefficients in each equation. If they are the same, then the lines will have the same slope, it will either have no solutions or infinite solutions. Look at the original system again:

$$2x + y = 5$$

$$4x + 2y = 10$$

Notice that the ratio of the coefficients, $\frac{-\mathrm{A}}{\mathrm{B}}$, for both equations is equal: $\frac{-2}{1} = \frac{-4}{2} = -2$. That means there are either no solutions or infinite solutions. The *y*-intercept will tell us which one, but remember that if the two equations are the exact same, there will be infinite solutions. Otherwise it will be no solutions.

If we divided the second equation by 2 on both sides we would get the first equation. Since the two equations would be the same, any point on the line represented by the first equation would be on the line of the second equation. That means we know there are infinite solutions and didn't have to do any work at all.

Now consider the following system. How many solutions are there?

$$3x - 2y = 5$$
$$2x - 3y = 5$$

Check the ratios of the coefficients. Notice that $\frac{-3}{-2} \neq \frac{-2}{-3}$ which means that the lines are not parallel. That tells us there is one solution, and we should use graphing, substitution, or elimination to find the solution.

Solution Steps

In essence, we follow these steps if the equations are not in slope-intercept form:

1) Make sure both equations are in standard form and check if the ratio of the coefficients are equal.
   a. If the ratios are not equal, there is a single solution, and you need to solve.
   b. If the ratios are equal, then check if you can make the equations exactly the same.
      i. If the equations can be made the same, there are infinite solutions.
      ii. If the equations cannot be made the same, there are no solutions.

Check Yourself: Decide if the following systems of equations have a single solution, no solutions, or infinite solutions. If it has a solution, solve the system.

1. $2x + y = 4$
   $y - 5 = -2x$

2. $x + \frac{1}{3}y = -10$
   $3x + y = -30$

3. $\frac{2}{3}x - y = 0$
   $2x = 3y$

4. $x + 4y = 2$
   $2(x + 4y) = 10$

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- [SOLVE SYSTEMS OF EQUATIONS THAT HAVE NO SOLUTION]
- [SOLVE SYSTEMS OF EQUATIONS WITH INFINITE SOLUTIONS]
- [SOLVE SYSTEMS OF EQUATIONS: USING SUBSTITUTION]
- [SOLVE SYSTEMS OF EQUATIONS: USING ELIMINATION]

TAKE THE **MAFS.8.EE.3.8** EDGENUITY PRE-TEST

YES

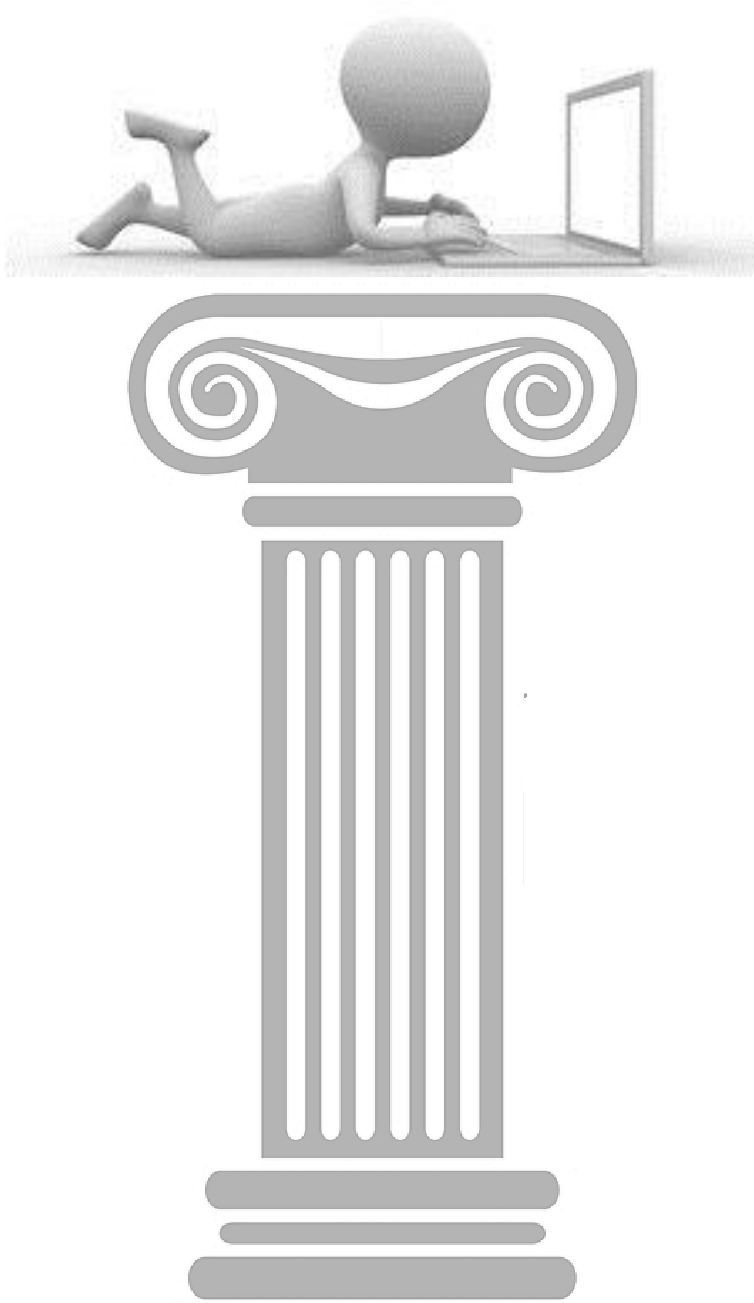
GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.EE.3.8**

I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

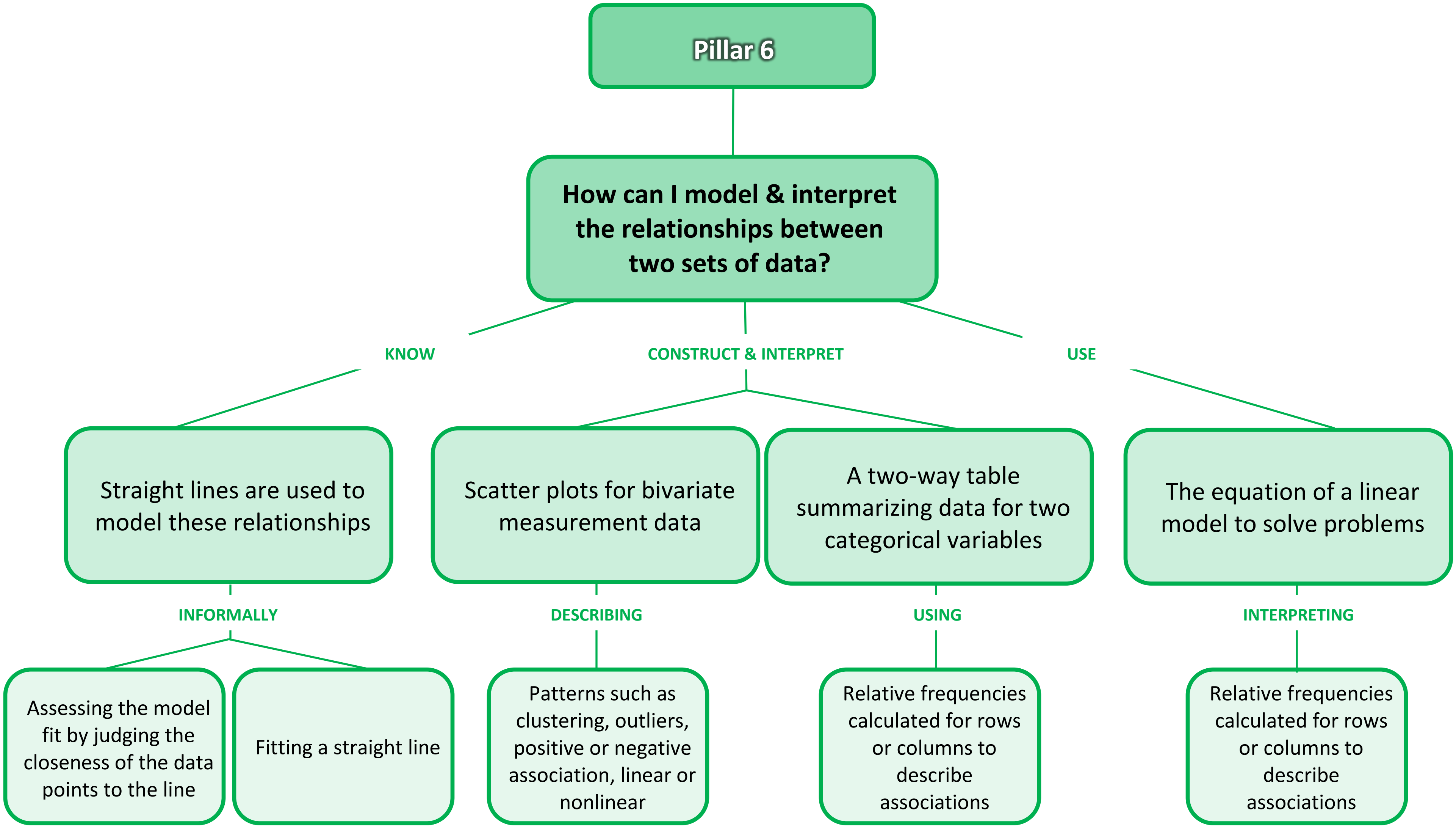
- COMPLETE LESSON **MAFS.8.EE.3.8** ON EDGENUITY
- GO TO LESSON 13: SCATTER PLOTS

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 13: SCATTER PLOTS

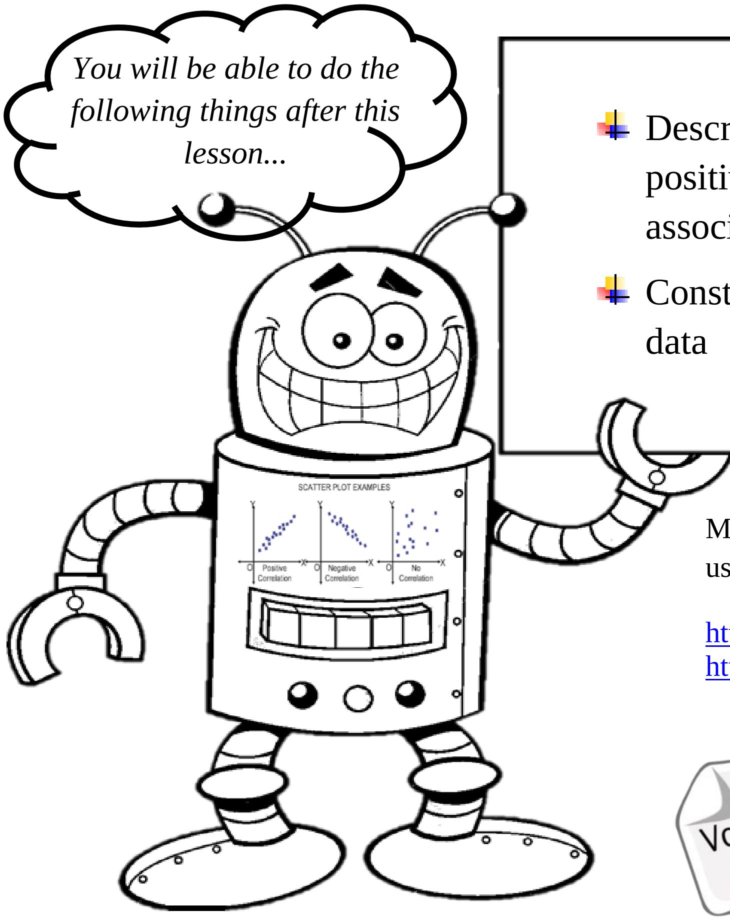


PILLAR #6



Algebra 1: Statistics » Interpreting Categorical & Quantitative Data

Lesson 13: Scatter Plots



- Describe patterns such as clustering, outliers, positive or negative association, and nonlinear association
- Construct scatter plots for bivariate measurement data

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- Bivariate
- Cluster
- Data
- Negative association/correlation
- Positive association/correlation

Constructing Scatter Plots

A **scatter plot** is a plot on the coordinate plane used to compare two sets of data and look for a correlation between those data sets. An **association** is a relationship or dependence between data. For example, the price of oil and the price of gasoline have a strong association. The daily price of oil and the number of penguins swimming in the ocean on that day most likely have no association at all. However, to find this association we need to make a scatter plot.

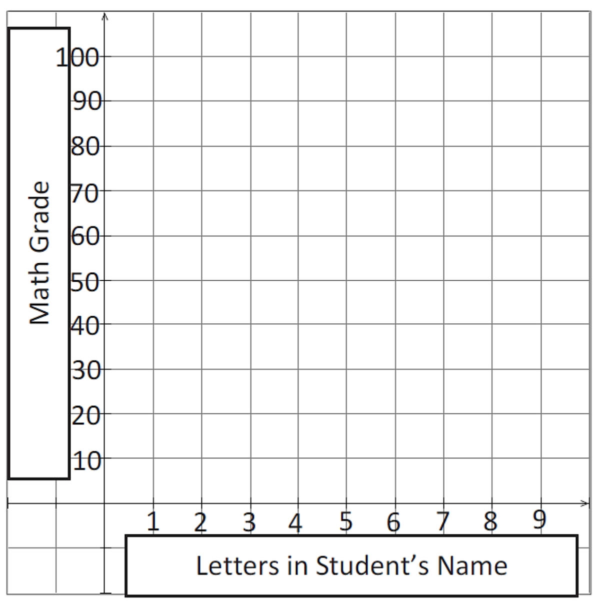
Start with the Data

Before we can make a scatter plot, we need two sets of data that we want to compare. For example, we might compare the number of letters in a student's first name and their math grade. Do people with shorter names tend to score higher in math? Do people with the lowest grades have longer names? These are questions of relationship, or correlation, that we can explore with a scatter plot once have some data. That data set might look like this:

Name	Nichole	Josiah	Kame	Gungar	Roberto	Frank	John	Herman	Sami	Daimon
Letters	7	6	4	6	7	5	4	6	4	6
Grade	58	83	61	70	31	76	81	70	72	57

Name	Volina	Johanne	Karoline	Kurt	Addison	Ian	Dennis	Ophelia	Kristina	Bradford
Letters	6	7	9	4	7	3	6	7	8	8
Grade	77	90	87	83	76	78	87	87	80	41

Prepare the Coordinate Plane



Now that we have our data, we need to decide how to put this data on the coordinate plane. We can let the *x*-axis be the number of letters in a student's name and the *y*-axis be the student's overall math grade. Once we have decided this we should label our axes.

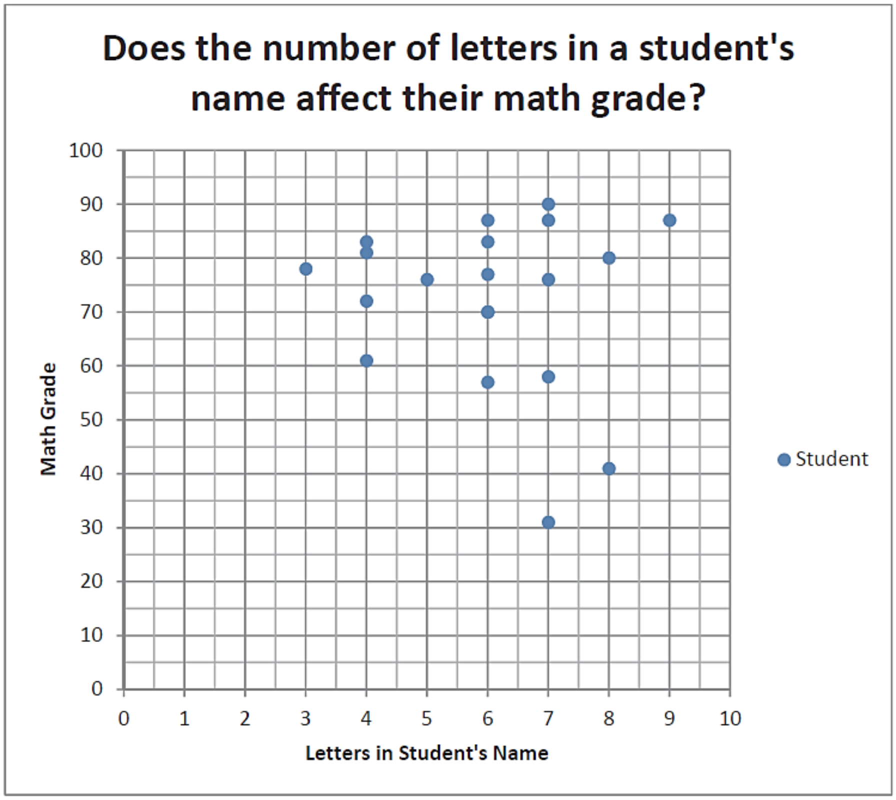
Next, we'll need to decide on a scale and interval. The scale is the low to high number on the axis and the interval is what we count by. Notice first that we're only looking at Quadrant I because we won't have negative amounts of letters or negative grades. Since the grades can be from zero to one hundred, we might choose to count by tens on the *y*-axis giving us a scale of 0-100 and an interval of 10. Since the letters range from three to nine, we might count by ones on the *x*-axis. This gives us a scale of 0-10 with an interval of 1.

When to use a broken axis

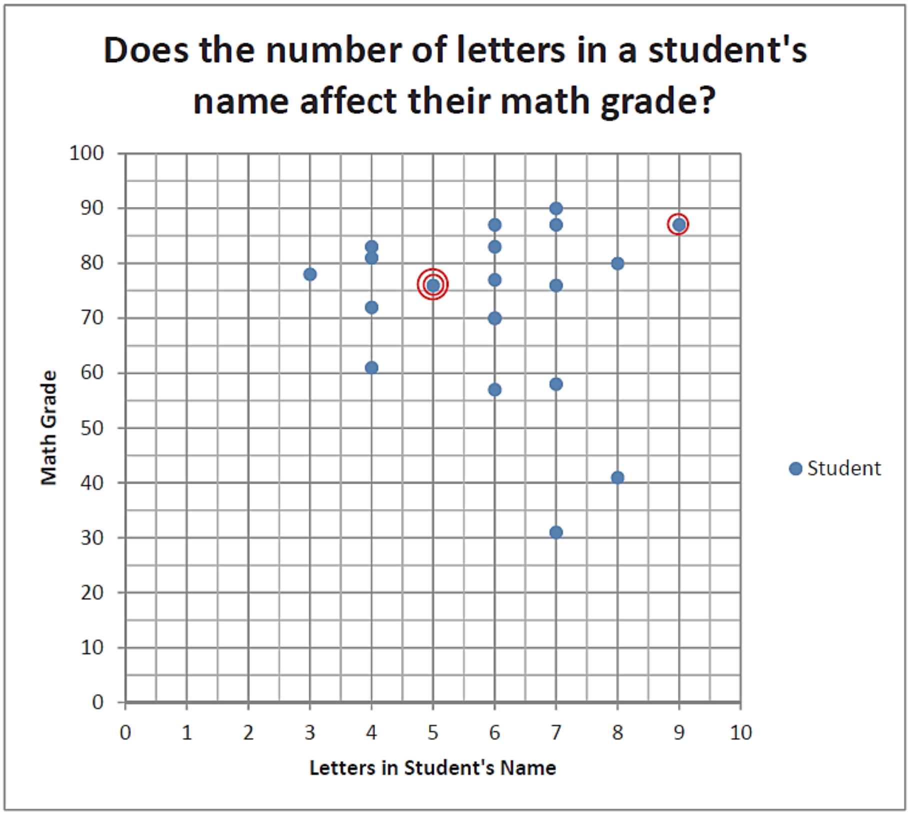
A broken axis is useful whenever more than half of the area of the scatter plot will be blank. Nobody likes to see a blank graph with all the data in one tiny area. So instead, we zoom in by using a broken axis. If the range of your data is less than the lowest data point, a broken axis may be useful. For example, in our math test situation above if everyone scored above a 60%, then we might break the *y*-axis and begin counting at 60. We could then count by 4's to make it up to 100%.

Plot the Points

Finally, we would then plot each person on the graph. So, Nicholas will be the point (7, 58), Josiah the point (6, 83), and so forth. Using Excel to make our scatter plot, the final scatter plot might look like the following. Notice that each dot on the graph represents a person. While the labeling is not necessary, it may be useful in some circumstances.



Many times, on a scatter plot you may have the same data point multiple times. One way to represent this fact is to put another circle around the data point. Let's add a few new students to our data set: Johnathan (9 letters and 87 math score), Jacob (5 letters and 76 math score), and Helga (5 letters and 76 math score). The new graph could look like this:



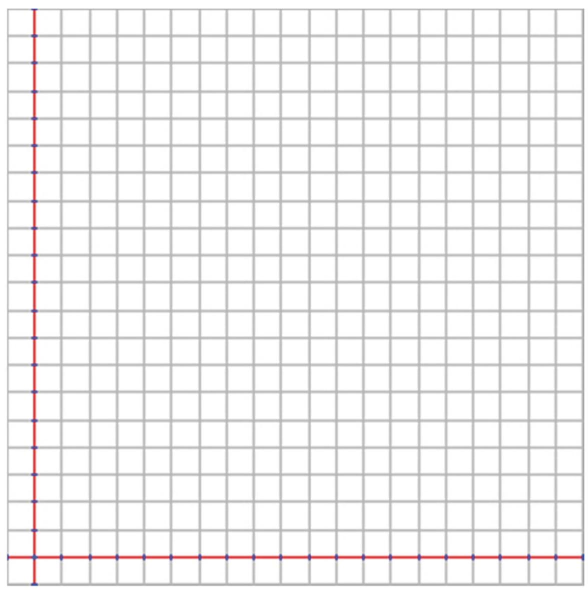
While this practice is not necessarily standard, it can be useful as a visual representation of what is happening with the data. We can more easily see the multiple data points this way.

Practice: Use the given data to answer the questions and construct the scatter plots.

Use the given data to answer the questions and construct the scatter plots.

Pathfinder Character Level vs. Total Experience Points

Level	2	3	6	9	10	11	14	15	17	20
XP	15	35	150	500	710	1050	2950	4250	8500	24000



1. Which variable should be the independent variable (x-axis) and which should be the dependent variable (y-axis)?

2. Should you use a broken axis? Why or why not?

3. What scale and interval should you use for the x-axis?

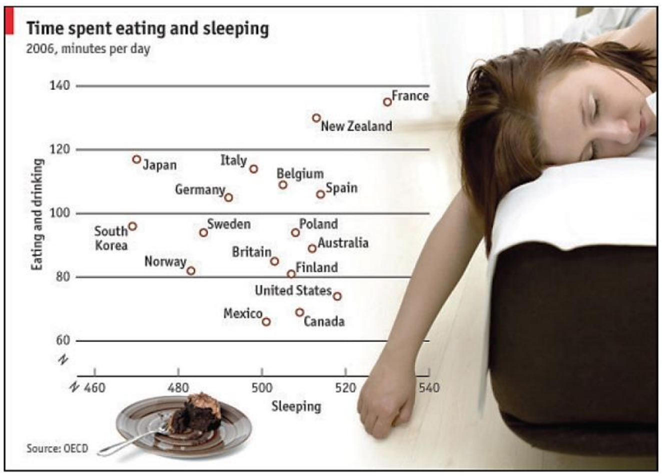
4. What scale and interval should you use for the y-axis?

5. Construct the scatter plot on the grid above.

Now that we know how to draw scatter plots, we need to know how to interpret them. A scatter plot graph can give us lots of important information about how data sets are related if we understand what each part of the graph means.

Reading Data Points

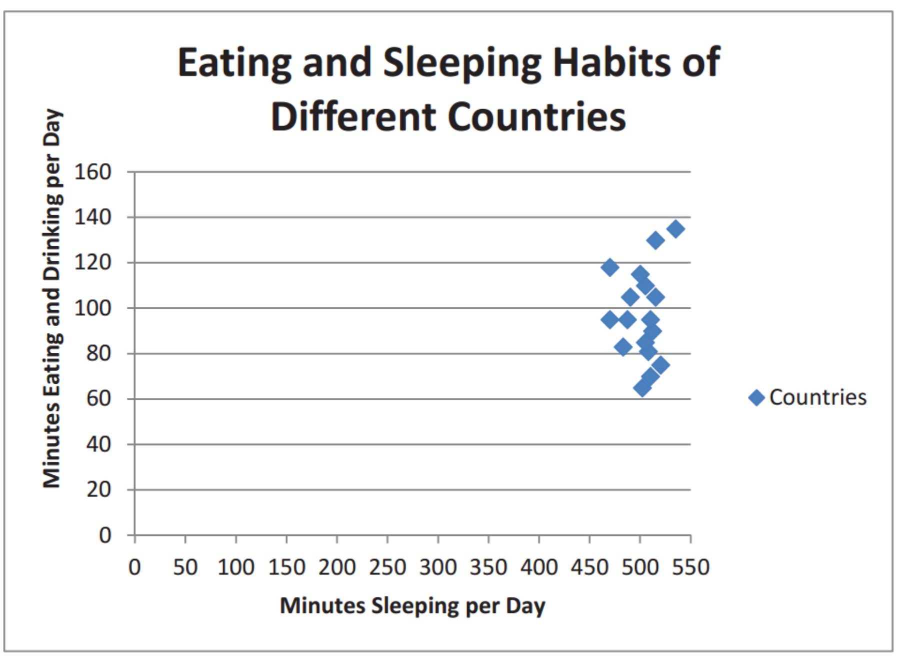
Each individual point on a scatter plot represents a single idea. For example, in the picture below each point represents a country. The axes tell us information about that country. The *y*-axis tells us about how many minutes per day that country spends eating and drinking. The *x*-axis tells us about how many minutes per day that country spends sleeping. Can you find the United States on this scatter plot? About how many minutes do we sleep per day? About how many minutes we spend eating and drinking per day? Are these numbers reasonable to you?



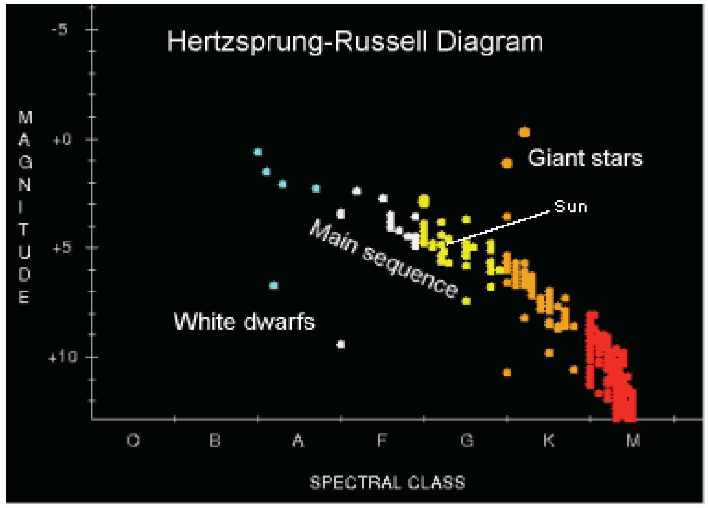
http://www.visualquest.in/2010/09/severalsimple-and-very-useful.html

Another thing to notice about this scatter plot is that it uses the broken axis symbol (that little Z looking thing). This means that they don't start counting from zero on either axis. They skip ahead to a reasonable starting point but still apply a scale after that point. Even with the broken axis they must count by something in each direction. In this case, they count by 20 minutes on the *x*-axis and the *y*-axis as well.

If we did not use the broken axis, it might look more like the scatter plot below. To be able to label the data points, it is useful in this case to use the broken axes.



Outliers



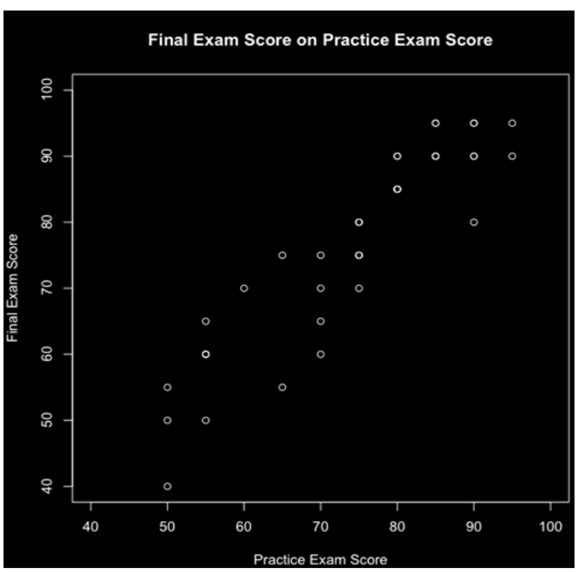
http://starplot.org/docs/ch1.htm

An outlier is a data point that is significantly far away from the majority of the data. There is no precise mathematical definition for what makes a data point an outlier. It's usually somewhat obvious. For example, notice that White Dwarf Stars and Giant Stars are both outliers in the below scatter plot showing a star's spectral class (temperature) versus its magnitude (brightness).

Why do we care about outliers? We care because outliers often throw off the analysis of the data set. For example, let's say you have three test grades in math class: 80%, 80%, and 80%. Your current class average is, you guessed it, 80%. However, if we throw in an outlier, like a 0%, for the next test, your class average drops down to 60%. You have dropped two letter grades from a B- to a D-. Yikes! The outlier sure hurt your grade.

Positive and Negative Associations

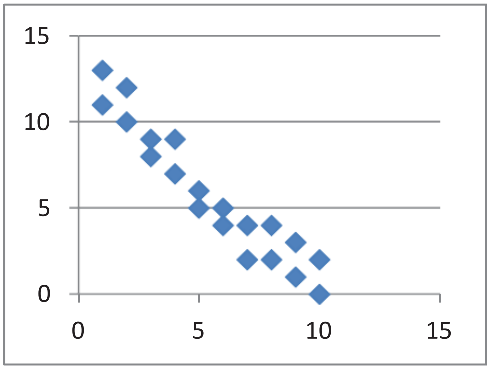
An association, sometimes called a correlation, is a relationship between two data sets. For example, in the above star scatter plot, there appears to be a relationship between a star's temperature and brightness. We'd have to know more about the science of stars to fully interpret the graph, but we can see there is an association because most of the data follows a pattern (except for those pesky outliers).



http://www.r-bloggers.com/r-tutorial-series-basic-polynomial-regression/

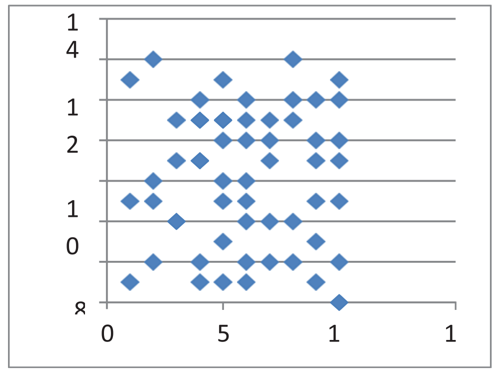
In fact, the more tightly clumped the data is, the stronger the association is. We might say that there is a strong association between the brightness and temperature of a star. In the scatter plot to the left, we see a slightly weaker association between scores on a practice exam and scores of the final exam.

We would also say that the scatter plot to the left has a positive association because it appears that the students who scored higher on the practice exam also scored higher on the final exam. As one variable (practice exam score) increased, the other variable (final exam score) also increased. We call this a positive association.



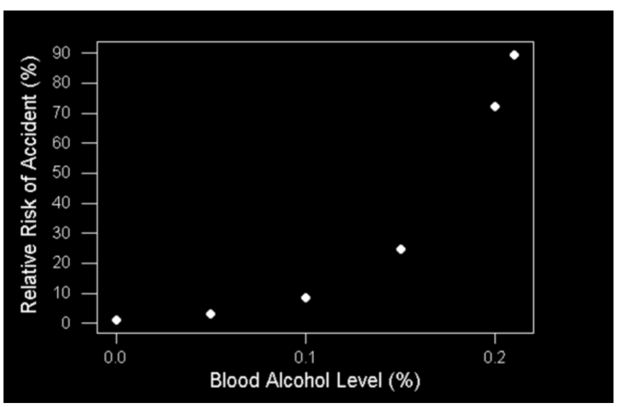
There are also negative associations. These associations are recognized by the fact that as one variable increases, the other decreases. For example, as the supply of oil increases, the cost of gasoline decreases. They have a negative association. A scatter plot with a negative association might look like the graph to the left.

No association would mean that there appears to be no relationship between the two data sets (or variables). For example, we might consider the daily price of tea and the daily number of fruit flies born. There is likely no relationship between those two things which would produce a graph similar to the one to the right.



Linear or Non-Linear Associations

Whether the association is positive or negative, it may appear linear or non-linear. A linear association would be a scatter plot where the data points clump together around what appears to be a line. The negative association graph above and to the left is an example of a linear association. The scatter plot about practice and final exams is an example of a positive linear association.

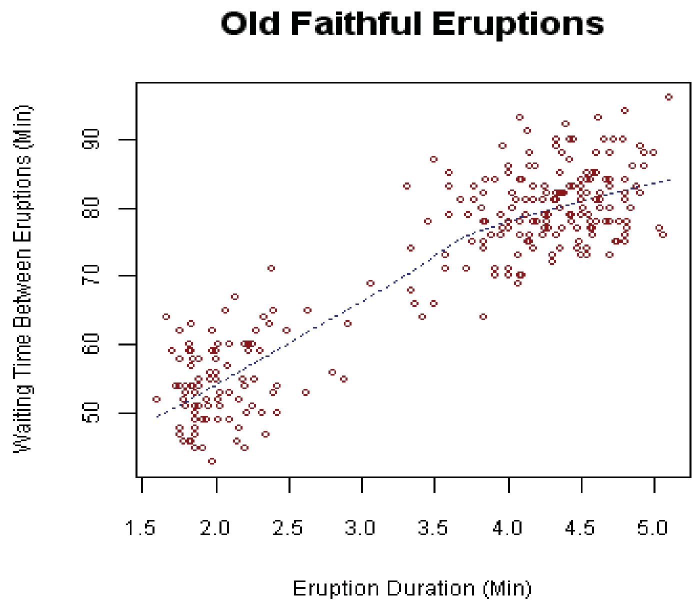


http://wps.prenhall.com/esm_walpole_probstats

A non-linear association is usually curved to some extent. There are many types of curves that it could fit, but we'll just focus on the fact that it doesn't look a line and therefore is non-linear. Consider the graph to the left showing the relative risk of an accident compared to the blood alcohol level. As you can see, the graph curves sharply up when there is more alcohol in the blood stream. This should not only serve as an example of non-linear scatter plot, but also the risks of drinking and driving.

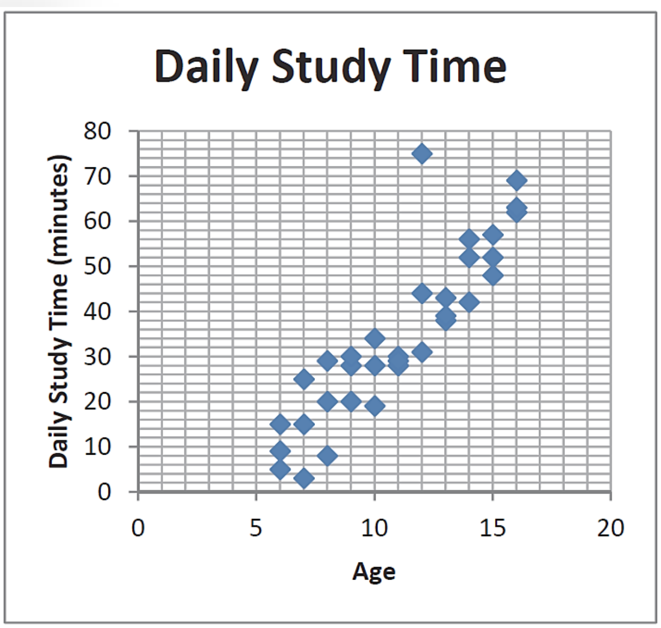
Clustering

Clustering is when there is an association, but it appears to come in clumps. Consider the following scatter plot that shows the time between eruptions and eruption duration of Old Faithful. Notice how the points cluster towards the lower left and upper right. While this does show us a positive association (meaning the longer between eruptions, the longer the next eruption will last), it also shows us that there are not very many medium length eruptions. They are either short eruptions with short wait times or long eruptions with long wait times.



http://en.wikipedia.org/wiki/Scatter_pl

Check Yourself: Use the given scatter plots to answer the questions.



1. Does this scatter plot show a positive association, negative association, or no association? Explain why.

2. Is there an outlier in this data set? If so, approximately how old is the outlier and about how many minutes does he or she study per day?

3. Is this association linear or non-linear? Explain why.

4. What can you say about the relationship between your age and the amount that you study?

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- CONSTRUCT A SCATTER PLOT
- INTERPRET A SCATTER PLOT BY IDENTIFYING CLUSTERS AND OUTLIERS
- INTERPRET AND DISTINGUISH LINEAR AND NONLINEAR SCATTER PLOTS

TAKE THE **MAFS.8.SP.1.1** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.SP.1.1**

I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

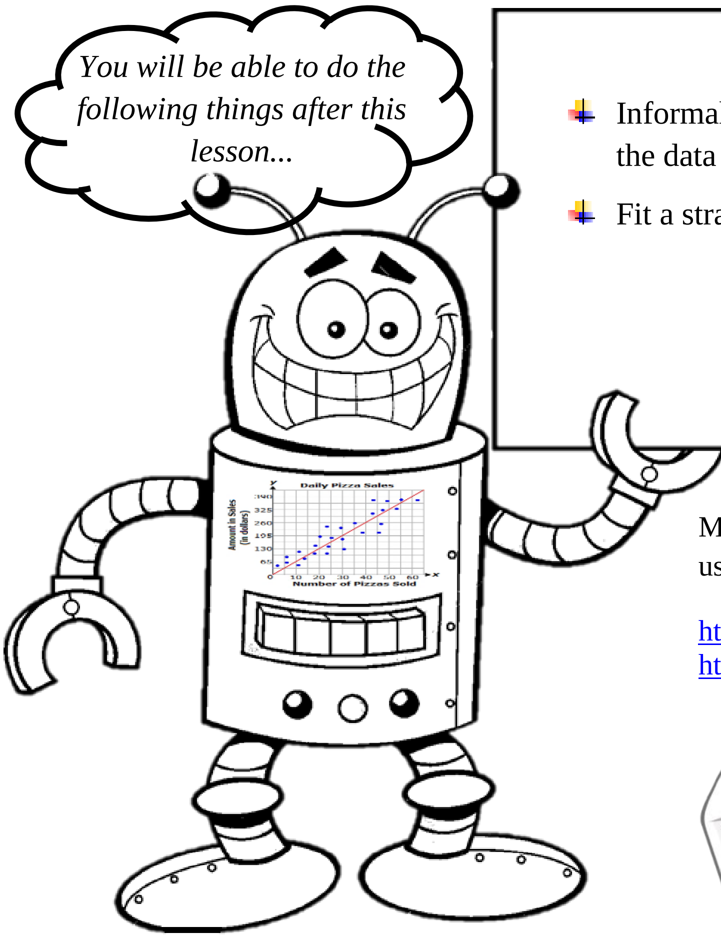
- COMPLETE LESSON **MAFS.8.SP.1.1** ON EDGENUITY
- GO TO LESSON 14: THE LINE OF BEST FIT

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 14: THE LINE OF BEST FIT

The resources in this packet have been compiled from the Regional Professional Development Program and Mr. Bright

Lesson 14: The Line of Best Fit



- Informally assess the model fit by judging the closeness of the data points to the line
- Fit a straight line within the plotted data

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- Initial value
- Line of best fit
- Trend line
- Linear association
- Extrapolate
- Interpolation

Line of Best Fit

When we have a scatter plot that suggests a linear association, it is often useful to draw in a line of best fit to help us interpret the data more accurately. A line of best fit is a line drawn on the scatter plot such that the distance between each of the points and the line are minimized. Let's look at some examples.

Drawing the Line of Best Fit

Finding the true line of best fit is quite an involved task if we do it by hand. While programs like Excel will automatically draw in the line of best fit for us, for now we will focus on informally drawing a line of best fit. In other words, we know that our line is not the exact line of best fit, but it will be a nice estimate. Consider the scatter plot to the right.

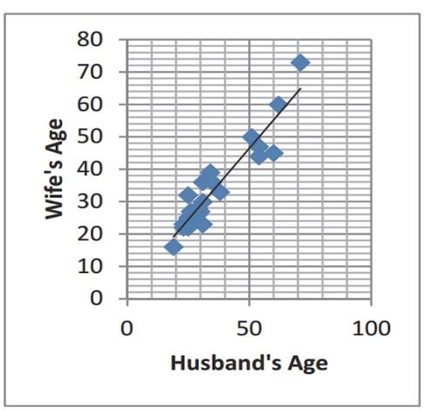
In this scatter plot there are 24 couples represented and it appears that there is a positive linear association between their ages. Generally speaking it looks like the older the husband is, the older the wife is. If we wanted to informally draw a line of best fit in this scatter plot, we would look for a place where we the line would roughly split data in half and have the same general rate of change (or slope) as the data.

Now consider the three scatter plots below. Which line of best fit seems most appropriate? The first attempted line of best fit does appear to cut the data roughly in half, but it definitely doesn't match the rate of change that the data seems to represent. The second attempted line of best fit seems to match the rate of change but doesn't roughly cut the data in half. The third one is our best option for an informal line of best fit.

First Attempt

Second Attempt

Third Attempt



Now for the sake of comparison, let's see the actual line of best fit that Excel comes up with. It looks like our line of best fit is very close to the true line of best fit.

Before drawing in the line of best fit on a given data set, it may be useful to lay down a pen or pencil on the scatter plot and try to arrange the pen where the line of best fit should be. Once you have visualized where the line of best fit should be, then draw it in.

Extrapolating with the Line of Best Fit

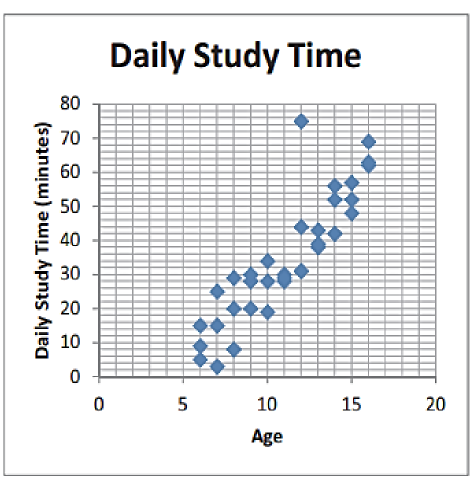
To extrapolate means to estimate or predict an answer in an unknown situation. We can use the line of best fit to make these predictions from the data. For example, using the above line of best fit, how old would we expect the wife to be of a husband that was 45 years old? We don't have a data point there, so we don't know what the answer to this would be, but we can extrapolate using the line of best fit. Go to 45 years old on the husband axis and go up to the line of best fit. Note that the line of best fit is at a height of 42 years old for the wife meaning this would be a good estimate for how old we would expect the wife to be.

If the wife were 60 years old, how old would we expect the husband to be? This time go to the height of 60 on the wife axis and travel over to the line of best fit. It appears to be at about 65 years old on the husband axis, so we would expect the husband to be near that age.

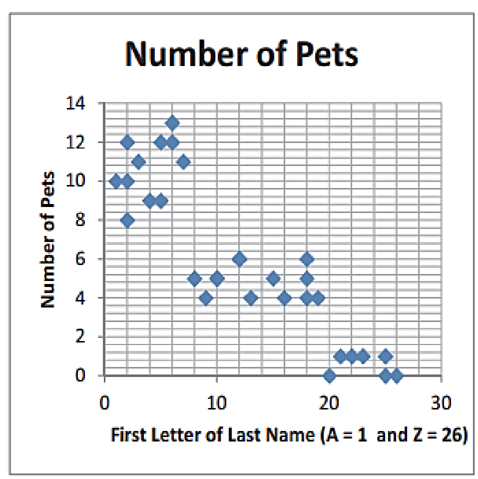
Notice that these are only estimates and would not necessarily be exactly what we would find in real life, but it is useful as a guideline.

Check Yourself: Draw a line of best fit on the given scatter plot.

1.



2.



DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- INTERPRET ASSOCIATION IN LINES OF BEST FIT
- INTERPRET THE SLOPE OF A LINE OF BEST FIT
- INTERPRET THE Y-INTERCEPT OF A LINE OF BEST FIT
- WRITE AN EQUATION FOR LINE OF BEST FIT
- USE THE EQUATION OF A LINE OF BEST FIT TO MAKE PREDICTIONS

TAKE THE **MAFS.8.SP.1.3** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.SP.1.3**

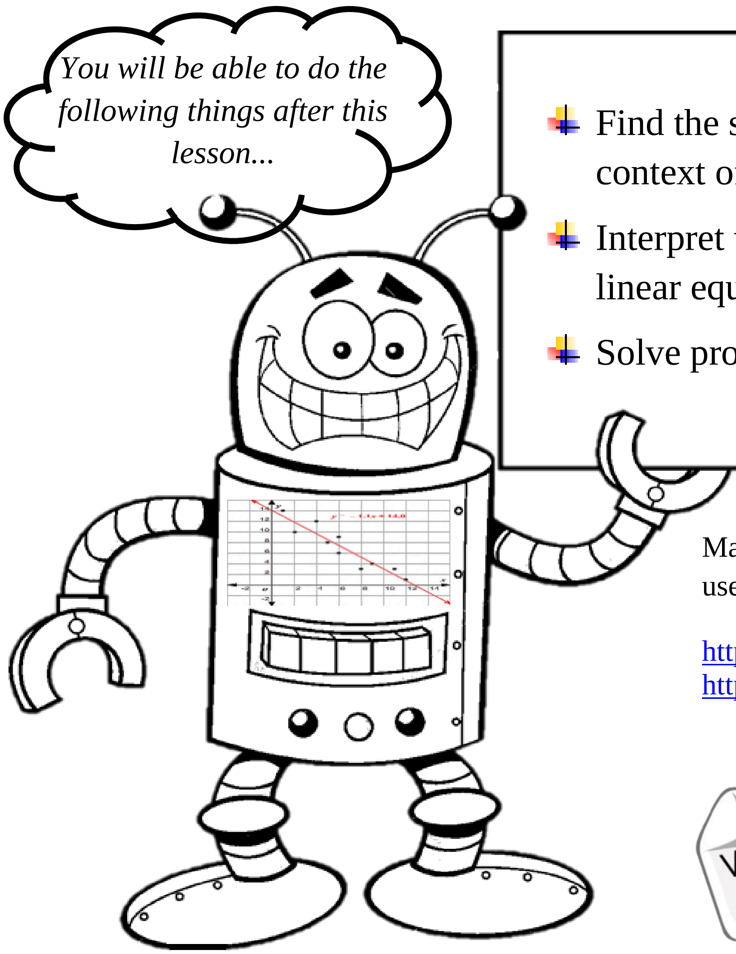
I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

- COMPLETE LESSON **MAFS.8.SP.1.3** ON EDGENUITY
- GO TO LESSON 15: PATTERNS OF ASSOCIATION

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 15: PATTERNS OF ASSOCIATION

Lesson 15: Patterns of Association



- Find the slope and intercept of a linear equation in the context of bivariate measurement data
- Interpret the meaning of the slope and intercept of a linear equation in terms of the situation
- Solve problems using the equation of a linear model

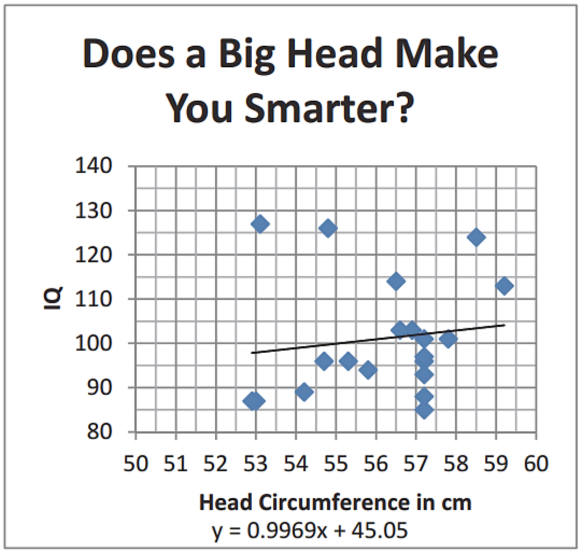
Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- Categorical data

The Equation of the Line of Best Fit

Since we have a *line* of best fit, we know that a line can be expressed as an equation. In fact, we are most familiar with the slope intercept form of an equation. We can use this line to extrapolate data further, learn more about the rate of change, and more. Let's look at some data taken from sets of twins where they were studying if there was an association between the size of a person's skull and his or her IQ.



First of all, notice that all the data is clustered between the 50 cm and 60 cm mark, so Excel decided it would be beneficial to use a broken axis on this graph. Secondly, notice that Excel has drawn in the line of best fit and given us the equation for that line.

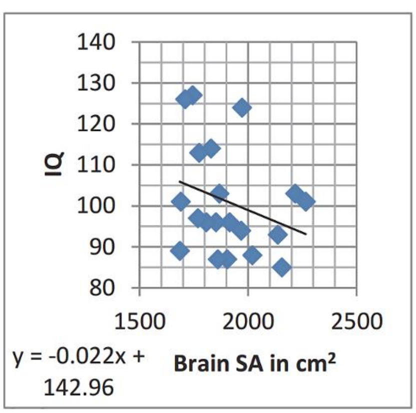
At first glance it appears that there may be no association between the size of your skull and your IQ. The line of best fit is nearly flat suggesting either a constant association or no association at all. However, because of the broken axes, this is misleading.

Let's first approximate the equation for ease of analysis. The slope of 0.9969 is very close 1 and the *y*-intercept is very close to 45, so let's approximate the line of best fit to be $y = x + 45$.

What does the slope mean in this context? The slope is approximately 1, which means that for one centimeter increase in skull size we would expect a one point increase in IQ. So maybe there is something to that old "egg head" comment, as mean as it is.

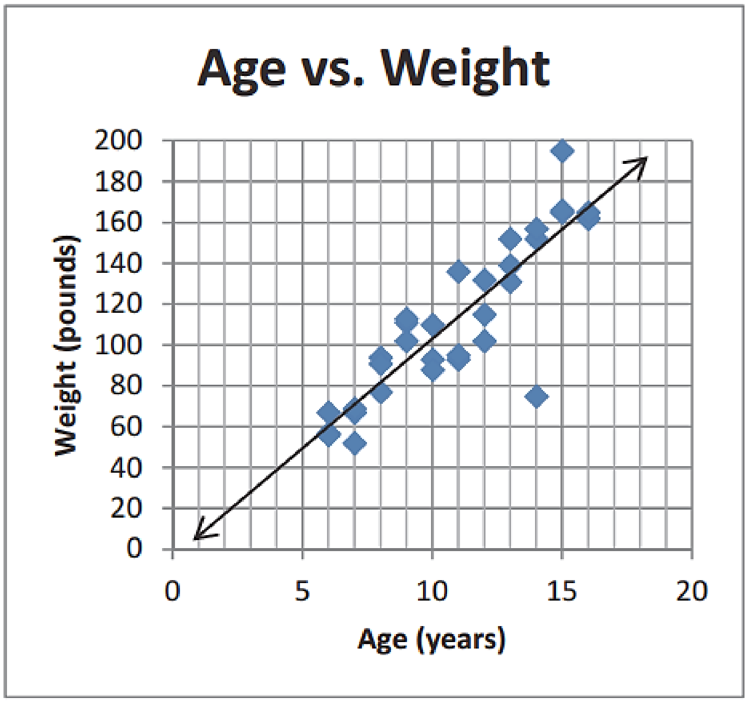
What does the *y*-intercept mean in this context? The *y*-intercept is about 45, which tells us that no matter the size of a person's head, their IQ is very unlikely to drop below 45. Even a skull size of zero centimeters in circumference would supposedly have an IQ of 45, but we know this isn't possible.

What would be the expected IQ if a person had a head circumference of 80 cm? In our equation, the y represents the IQ and *x* represents the head circumference. Simply plug in and solve like this: $y = 80 + 45 = 125$ to see that the expected IQ would be about 125. If a person had an IQ of 150, what would we expect their head circumference to be using our line of best fit? $150 = x + 45$ and then subtract 45 from both sides to see that $x = 115$ cm. That's a big head!



This final graph shows the surface area of the brain compared to IQ. If we rounded the slope and intercept, the equation of the line of best fit is approximately $y = \frac{1}{50}x + 143$. This means that the IQ goes down one for every additional 50 cm^2 in surface area. In context that means that the more of your brain that is "exposed", the lower your IQ.

Check Yourself: Use the given line of best fit or equation of the line of best fit to answer the following questions.



1. Using the graph only, about how much would you expect an 18-year-old to weigh?

2. Using the graph only, about how much would you expect a 4-year-old to weigh?

3. Using the graph only, if a person weighed 120 pounds, how old would you expect them to be?

4. The line of best fit for the scatter plot showing age (x-value) compared to weight (y-value) is approximately: $y = \frac{21}{2}x - \frac{7}{2}$. Using the line of best fit equation (show your work), about how much would you expect an 18-year-old to weigh? How does this answer compare to the answer for problem 1?

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- [DRAW A LINE OF BEST FIT]()

TAKE THE **MAFS.8.SP.1.3** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.SP.1.3**

I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

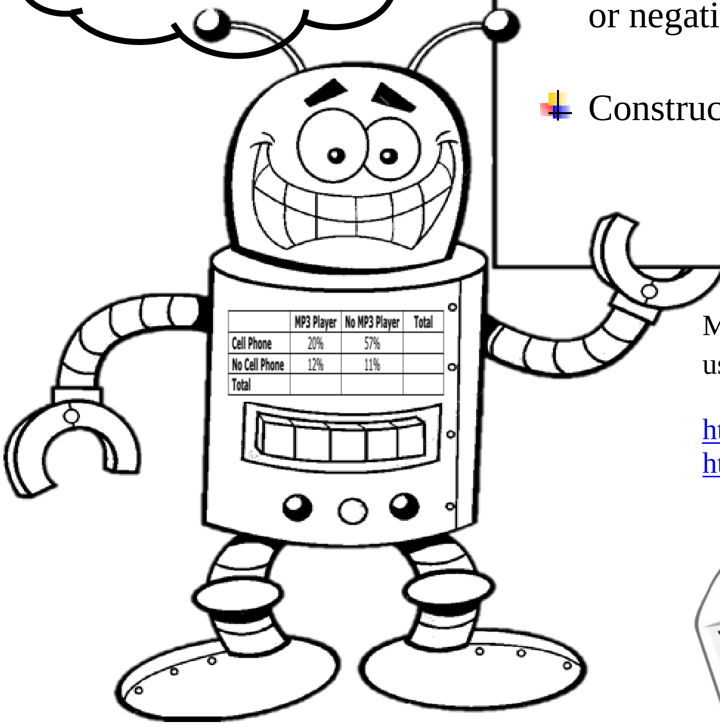
- COMPLETE LESSON **MAFS.8.SP.1.3** ON EDGENUITY
- GO TO LESSON 16: TWO-WAY TABLES

I SCORED 70% OR ABOVE ON THE EDGENUITY PRE-TEST

- GO TO LESSON 16: TWO-WAY TABLES

Lesson 16: Two-Way Tables

You will be able to do the following things after this lesson...



- Describe patterns such as clustering, outliers, positive or negative association, and nonlinear association
- Construct scatter plots for bivariate measurement data

Math is Fun and Math Words are suggested sites that can be used to define the vocabulary words for this lesson:

https://www.mathsisfun.com/definitions/index.html
http://www.mathwords.com/

- Bivariate
- Cluster
- Data
- Negative association/correlation
- Positive association/correlation

Two-Way Tables

Sometimes we need to compare two sets of data where the data is a yes/no type answer. In this case a scatter plot doesn't make sense since we don't have numerical data. We use what is called a two-way table to analyze this type of data.

Constructing a Two-Way Table

To construct a two-way table, we first need some data. Let's look at the following fictional table where we asked a class of 22 students to choose one the fictional political parties: Champions or Stars, and then answer a series of questions:

	Anne	Brad	Cathy	Devin	Edith	Frank	Gabby	Hannah	Ignus	Jake	Koty
Champions/ Stars?	C	S	S	S	S	S	S	S	S	C	C
Do you eat McDonald's weekly?	y	y	y	N	N	y	y	y	y	N	N
Want higher taxes?	y	N	N	N	N	y	N	N	N	y	N
Do you own smartphone?	N	y	y	N	y	y	N	y	y	N	N

	Lisa	Mo	Nancy	Opy	Peggy	Quira	Ron	Shela	Toni	Ula	Vanna
Champions/ Stars?	C	C	C	S	S	C	S	C	S	C	C
Do you eat McDonald's weekly?	N	y	N	y	N	N	N	N	y	N	N
Want higher taxes?	N	y	y	N	N	N	y	N	N	y	y
Do you own smartphone?	y	y	y	N	y	y	y	y	y	y	y

Now that we have our data, we will consolidate some of it into a two-way table. Let's first compare Happyville’s political view between Champions and Stars to their eating habits at McDonald's. A two-way table for this comparison would look like this:

	Champions	Stars
Eat McD's weekly	3	7
Don't eat McD's weekly	9	3

How did we fill this out? We counted the number of Champions that eat at McDonald's weekly, the number of Stars that eat at McDonald's weekly, the number of Champions that don't eat at McDonald's weekly, and the number of Stars that don’t eat at McDonald’s weekly. Each of those numbers we filled in the table in the appropriate place. Obviously one of the advantages of the two-way table is the fact that it takes up so much less space than the original data. We could make a similar two-way table comparing political affiliation with tax views or comparing tax views with owning a smart phone.

Analyzing a Two-Way Table

There are many things that a two-way table can tell us. Let's look at another example of how the Happyville voted on a recent bill that would force the national budget to be balanced.

	Champions	Stars
In favor	25	236
Against	161	4

How many Champions voted on this bill? If 25 voted in favor of the bill and 161 voted against, that means that a total of 186 Champions voted on this bill. In essence we are finding the frequency of being a Champion by adding the numbers in the Champion column. Frequency is how often something occurs.

Similarly, we can see how many Stars voted on this bill, which is 240. How many total representatives voted on this bill? We can find this by adding all the numbers together. This means that 426 representatives in total voted on this bill. Since there are 435 representatives (we know this from our social studies class), we can then ask why the remaining 9 representatives didn't vote. Call them and ask.

We can also see that 261 voted in favor of the bill and 165 voted against the bill by adding the numbers in the rows. While this is a majority vote, it is not the required 290 votes needed to pass, so ultimately this bill failed.

At times, it may be more useful to look at the relative frequency instead of the frequency. **Relative frequency** is the ratio of the frequency to the total number of data entries. So, while the frequency of in favor votes was 261, it might be more useful to know that the relative frequency is $\frac{261}{426} \approx 0.61$. So about 61% of the House voted in favor of this bill and a vote of $\frac{2}{3}$ or $66.\overline{6}\%$ was needed for the bill to pass.

Are there any other conclusions we can make based on the information in the two-way table? For example, is there evidence that one party supported the bill over the other? It appears from the data table that there is a positive association between being a Star and being in favor of the balanced budget bill. It appears that there is a negative association between being a Champion and being in favor of the balanced budget bill. Notice that this doesn't mean that the Stars are positive (or correct) and Champions are negative (or wrong). Instead the positive and negative refer to the association or correlation in the data.

Check Yourself: Use the data set to answer the following questions. For this data set a class of middle school students was asked what they thought was most important in school: good grades or popularity.

Boy or Girl	B	B	G	G	G	B	G	B	B	G	G	B	G	B	G	B	B	G	G	B
Grades or Popularity	P	G	G	P	G	P	G	G	P	G	G	P	G	P	P	P	G	G	G	P

Boy or Girl	B	B	G	G	G	B	G	B	B	G	G	B	G	B	G	B	B	G	G	B
Grades or Popularity	P	G	P	G	G	P	G	P	P	G	G	G	G	P	P	P	G	P	G	G

1. Construct a two-way table of the data.

	Grades	Popularity	Total
Boys			
Girls			
Total			

2. How many students believe grades are important?

3. How many girls believe grades are important?

4. How many more girls believe popularity is more important than boys?

DO YOU THINK YOU ARE READY TO MOVE ON?

NO

GO TO THE WEBSITES BELOW FOR ADDITIONAL ASSITANCE:

- [CONSTRUCT A TWO-WAY TABLE FROM A LIST]
- [CONSTRUCT A TWO-WAY TABLE BY INTERPRETING A VENN DIAGRAM]

TAKE THE **MAFS.8.SP.1.4** EDGENUITY PRE-TEST

YES

GO TO Edgenuity AND COMPLETE THE PRE-TEST FOR **MAFS.8.SP.1.4**

I SCORED BELOW 70% ON THE EDGENUITY PRE-TEST

- COMPLETE LESSON **MAFS.8.SP.1.4** ON EDGENUITY

I SCORED 70% OR PILLAR #1THE EDGENUITY PRE-TEST

- DO A HAPPY DANCE, YOU ARE DONE!

The resources in this packet have been compiled from the Regional Professional Development Program and Mr. Bright